UNIT-I

1. Electrostatics and Dielectrics.

Gauss's Law statement and its proof:-

Gauss's Law states that the total normal electric flux ϕ_E over a closed surface is $\frac{1}{\epsilon_0} q \rightarrow (1)$ the total charge q enclosed with in the surface.

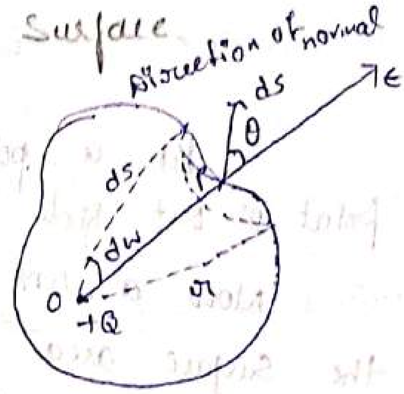
It can be expressed as

$$\phi_E = \oint E\,ds = \oint E \cos\theta\, ds = \frac{1}{\epsilon_0}(q) \rightarrow (1)$$

where ϵ_0 is permittivity of the free space.

Proof: 1. when the charge is with in the surface

Let charge $+Q$ is placed at 'O' with in a closed surface of irregular space as shown in fig consider a point 'p' on the surface at a distance 'r' from 'O'.



Now take a small area ds around 'p'. Now it is making an angle of θ with direction of electric field E along op.

The electric flux $d\phi_E$ through area 'ds' is given by

$$d\phi_E = E.ds = E.ds\cos\theta \rightarrow (2)$$

from columbs Law $E = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r^2}\right) \rightarrow (3)$

from (2) & (3)

$$d\phi_E = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r^2}\right)\cos\theta\, ds$$

$$d\phi_E = \frac{Q}{4\pi\epsilon_0}\left(\frac{\cos\theta\, ds}{r^2}\right)$$

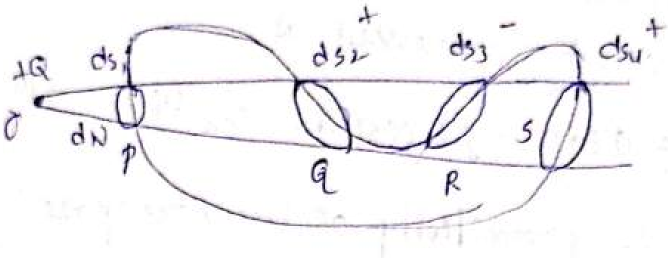
$$d\phi_E = \frac{Q}{4\pi\epsilon_0}\, dw \rightarrow (4)$$

where $\oint d\omega$ is equal to solid angle which is equal to 4π then $\oint d\phi_E = \frac{Q}{4\pi\varepsilon_0} \oint d\omega$

$$\phi_E = \frac{Q}{4\pi\varepsilon_0} \times 4\pi$$

$$\phi_E = \frac{Q}{\varepsilon_0} \rightarrow (5)$$

(2) when the charge is out side the surface:-



Let a point charge $+Q$ be situated let a point 'O' out side the closed surface as shown in fig.

Now a cone of solid angle $d\omega$ from 'O' cut the surface area ds_1, ds_2, ds_3 and ds_4 at points P, Q, R, S

The electric flux for an outward normal is positive (+ve) while inward normal is negative (-ve) so the flux through that area ds_2 and ds_4 are +ve and ds_1 and ds_3 are -ve.

the electric flux at P through the area $ds_1 = -\left(\frac{Q}{4\pi\varepsilon_0}\right) d\omega$

the electric flux at Q through the area $ds_2 = \left(\frac{Q}{4\pi\varepsilon_0}\right) d\omega$

the electric flux at R through the area $ds_3 = -\left(\frac{Q}{4\pi\varepsilon_0}\right) d\omega$

the electric flux at S through the area $ds_4 = \left(\frac{Q}{4\pi\varepsilon_0}\right) d\omega$

the total flux ϕ_E is

$$\phi_E = -\left(\frac{Q}{4\pi\varepsilon_0}\right) d\omega + \left(\frac{Q}{4\pi\varepsilon_0}\right) d\omega - \left(\frac{Q}{4\pi\varepsilon_0}\right) d\omega + \left(\frac{Q}{4\pi\varepsilon_0}\right) d\omega$$

$$\phi_E = 0$$

so that the electric flux over the whole surface due to an external charge is '0'.

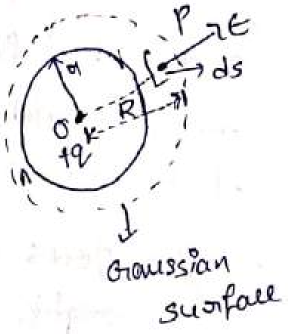
this verifies gauss's law.

⇒ Electric field intensity due to

(i) uniformly charged sphere

(ii) an infinite conducting sheet of charge:-

(i) uniformly charged sphere:-



Consider a sphere of radius r with centre O as shown in figure. Let a charge q be uniformly distributed over it. Here P is external point and at that point we have to calculate electric field. Let us construct a Gaussian surface of radius OP with sphere A.

Let ds be the small gaussian surface, then

$E \cdot ds = E\,ds \cos 0° = E\,ds$ {angle E} ds

Now the electric flux through the entire Gaussian surface is given by

$$\phi_E = \oint E \cdot ds = E \oint ds = E(4\pi r^2) \longrightarrow ①$$

According to Gauss's Law the total flux over a closed surface is $\left(\frac{1}{\epsilon_0}\right)$ times the charge enclosed with in the surface, Thus

$$E \cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

3

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \quad \text{Newton/columb}$$

In vector form

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^3} \quad \text{Newton/columb}$$

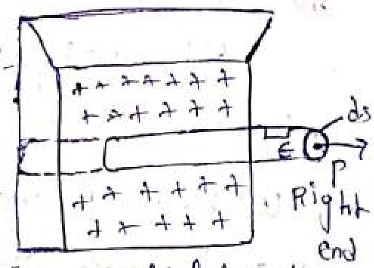
When p lies on the surface of the sphere, then r=R. In this case the intensity is given by.

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2} \quad \text{newton/columb}$$

Scanned with OKEN Scanner

(ii) An Infinite conducting sheet of charge:-

The fig shows a charged conducting surface of charge density σ. we have to determine the electric field intensity at a point P.



From figure At right end E is parallel to ds and at left end no electric field. The flux through the two ends are Eds and zero respectively. The flux are ⊥er. Therefore

$$\phi = \oint_{\substack{\text{right}\\\text{end}}} E\,ds + \oint_{\substack{\text{left}\\\text{end}}} E\cdot ds + \oint_{\text{curved surface}} E\cdot ds$$

(or)

$$\phi = Es + 0 + 0 = Es \longrightarrow (1)$$

According to Gauss's Law

$$Es = \frac{q}{\epsilon_0} = \frac{\sigma s}{\epsilon_0} \qquad \left[\frac{q}{s} = \sigma\right]$$

$$\boxed{E = \frac{\sigma}{\epsilon_0}} \longrightarrow (2)$$

⇒ Electric potential:-

consider an electric field due to +ve charge +q as shown in fig. Let there be a +ve test charge $+q_0$ in this field. if the test charge $+q_0$ moves from point B to another point A, then some work has to be done against the force of repulsion.

The ratio of workdone in taking a test charge from one point to other point in an electric field the magnitude of the test charge q_0 is defined as the electric potential difference b/w the points.

If 'W' be the workdone in moving test charge q_0 from B point to A then the potential difference $(V_A - V_B)$ b/w A and B is expressed as

$$V_A - V_B = \frac{W}{q_0} \longrightarrow (1)$$

if 'B' is at infinity then $V_A = \frac{W}{q_0} \longrightarrow (2)$

Thus the electric potential at a point in the electric field is defined as workdone by an external agent in carrying a unit +ve test charge from infinity to that point against the electric force of the field.

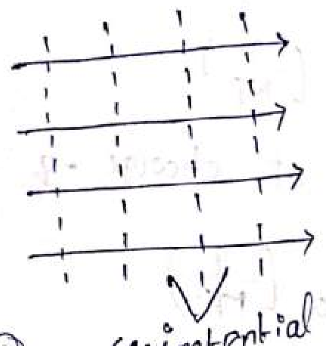
P.D - M.K.S - volts :- 1 Joule / 1 columb.

⇒ equipotential surface :-

Equipotential surface is an electric field is a surface on which the potential is same at every point. the locus of all points which have the same electric potential is called equipotential surface.

The lines of force at every point of the equipotential surface perpendicular to the surface.

8



(a)

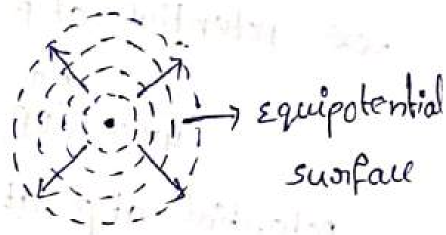


fig (b)

In the case of uniform field, where the lines of forces are straight and parallel, the equipotential surfaces are planes $\perp^{er}$ to the lines of force as shown in fig (a) and fig (b).

Equipotential

⇒ Potential due to (i) Dipole, (ii) uniformly charged sphere:-

(i) Dipole:- The arrangement of two equal and opposite point charge at fixed distance is called an electric dipole.

6

When two charges equal in magnitude but opposite in sign are seperated by a small distance the system is called an electric dipole.

As shown in fig the dipole in charges +q and -q of length 2a we have two determined the potential at external point P at a distance D from the centre 'O'.

Let BM and AM are perpendicular to OP.

Now potential at P at B. point of charge +q

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{BP}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{NP}\right)$$

potential at P at A point of charge -q

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{AP}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{MP}\right)$$

∴ potential diff at P

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{NP} - \frac{q}{MP}\right) \longrightarrow (1)$$

From $\Delta ONB = ON = OB\cos\theta = a\cos\theta$

$$NP = OP - ON = (r - a\cos\theta)$$

Illy ΔAMO, $OM = AO\cos\theta = a\cos\theta$

$MP = MO + OP = (a\cos\theta + r) = (r + a\cos\theta)$

substitute in Eqn ①

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r - a\cos\theta} - \frac{q}{r + a\cos\theta}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{(r + a\cos\theta) - (r - a\cos\theta)}{(r - a\cos\theta) - (r + a\cos\theta)}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{2a\cos\theta}{r^2 - a^2\cos^2\theta}\right] = \left(\frac{2aq\cos\theta}{4\pi\varepsilon_0 (r^2 - a^2\cos^2\theta)}\right)$$

$$V_1 = \frac{p\cos\theta}{4\pi\varepsilon_0 (r^2 a^2\cos^2\theta)} \longrightarrow ②$$

As $r^2 >> a^2$, $a^2\cos^2\theta$ is neglected

Hence $V = \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2} \longrightarrow ③$ //

If $\theta = 90^\circ$ that is equitorial plane $B = 0$

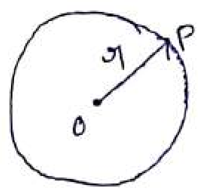
Thus the potential vanishes every where in the equatorial planee.

So we conclude that

(1) The equitorial plane is equipotential line moment.

(2) The potential of a dipole depends upon dipole moment 'p' and not q or $2a$ superated.

(ii) uniformuly charged sphere :-



consider a Gaussian surface which is a sphere of radius 'r',

From the Gauss Law

$$A(r)\, E_r(r) = \frac{q(r)}{\varepsilon_0} \longrightarrow ①$$

where $A_r = 4\pi r^2$ area of the surface of the sphere

E_r = The radial electric field strength at radius 'r'

And $q(r)$ = Total charge enclosed by the surface

then $q(r) = Q_1 \quad r > a$

$$= Q\left(\frac{r}{a^3}\right), \quad r < a$$

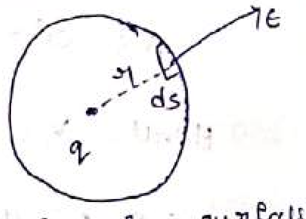
Thus $$E(r) = \frac{Q}{4\pi\epsilon_0 r} \quad r > a \text{ outside}$$

$$= \frac{Q}{4\pi\epsilon_0 a^3} \quad r < a \text{ inside}$$

From above it is clearly said that, electric field strength is proportional to small 'r' inside the sphere but false off like $\frac{1}{r^2}$ outside the sphere.

Deduction of columb's Law from Gauss's Law :-

Consider an isolated point charge 'q' as shown in figure.



Gaussian surface

Construct a Gaussian surface of radius 'r' at any point on the spherical surface the electric field intensity (E)

. According to Gauss' Law the total electric flux over a spherical surface is given by

$$\oint E ds = \frac{q}{\epsilon_0} \longrightarrow (1)$$

$$\epsilon_0 \oint E \cdot ds = q$$

$$\epsilon_0 E \cdot \oint ds = q$$

$$\epsilon_0 E \left[4\pi r^2\right] = q$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \longrightarrow (2)$$

If at a spherical surface, a second charge q_0 is placed with experiences a force is given by $F = E \cdot q_0 \longrightarrow (3)$

$$F = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \cdot q_0$$

From equation (2)

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q q_0}{r^2} \longrightarrow (4)$$

This is columb's Law.

9

2. Dielectrics

10

⇒ **Polar and non polar dielectrics in an electric field**

Non-polar dielectric:- When a dielectric slab is placed in an electric field between the plates of charged condenser, the +ve and -ve charges are reoriented.

Thus the net effect of the applied field is to seperate the +ve charges from the -ve charges. This effect is known as polaraisation of dielectrics.

The dielectrics which are polaraised only when they are placed in an electric field are called non-polar dielectrics.

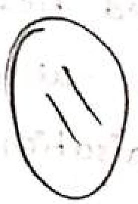
Fig ⓐ shows the random distribution of plus and minus charged in a non-polar dielectric. When this dielectric is placed in an electric field E_0, surface charges appear as shown in fig ⓑ. As shown in fig ⓒ the induced surface charges appear in such a way that the electric field set up by them E' opposes the external electric field E_0. The resultant field E in the dielectric is the vector sum of E' and E_0

$$\therefore \boxed{E = E_0 + E'}$$

Thus, if the dielectric is placed in an electric field, induced surface charges appear which tend to weaken the original field within the dielectric.

Polar dielectric in electric field:-

We know that polar dielectrics have permanent dipole moments with their random orientation as shown in figure (a).



In the presence of an electric field the partial alignment of dipoles takes place as shown in fig (b). The alignment increases with the increase of electric field or with the decrease of temparature. The dipole moment of a polar molecule in an electric field will be $P_p + P_i$. where P_p is permanent dipole moment and P_i is induced dipole moment.

Thus the non polar molecules in an electric field become induced dipoles while polar molecules are re-oriented with dipole moment increased.

Dielectric constant and susceptability:-

⇒ Dielectric constant

Dielectric constant:- When a dielectric is placed between ratio of a condensor its capacity is increased. The ratio of a condensor with dielectric to the capacitance of the same condenser without dielectric is defines as dielectric constant. Thus

$$K = \frac{C \text{ (capacitance of condenser with dielectric)}}{C_0 \text{ (capacitance of condenser without dielectric)}} \longrightarrow (1)$$

The ratio of potential difference without dielectric to the potential difference with dielectric is defined as dielectric constance. hence

$$\frac{V_0}{V_d} = K \longrightarrow (2)$$

12 Dielectric constant is a ratio. Its value is one for vacuum and ∞ for metals.

Electric susceptability:-

When a dielectric is placed in electric field it is polarised the polarisation vector P is proportional to the electric field E.

hence $P \propto E \longrightarrow (3)$

(or) $P = \in E \longrightarrow (4)$

where the constant of proportionality ∈ is known as electric susceptability.

The electric susceptibility way be defined as the ratio of polaraization vector to the electric intensity in the dielectric.

capacitance of a parallel plate capacitor with dielectric slab b/w the plates.

Fig shows a capacitor partially filled with a dielectric material. Let A and B be the plates of the capacitor with area A: seperated by a distance d. The upper plate A is given a charge +q while the lower plate B is given a charge -q. Now the dielectric slab with thickness t and dielectric constant K integer between the plates. consider the electric field E_0 and E in air and dielectric respectively.

(13)

We consider Gaussian surfaces PQRS and P'Q'R'S' as shown in fig.

$$\int E_0 ds = \frac{q}{\epsilon_0} \quad \text{or} \quad E_0 A = \frac{q}{\epsilon_0}$$

$$\therefore E_0 = \frac{q}{\epsilon_0 A} \longrightarrow (1)$$

Ily for the Gaussian surface P'Q'R'S', we have

$$\int K \cdot E \cdot ds = \frac{q}{\epsilon_0} \quad (\text{or}) \quad K \cdot E \cdot A = \frac{q}{\epsilon_0}$$

$$\therefore E = \frac{q}{K \epsilon_0 A} \longrightarrow (2)$$

The potential difference V between the two plates is the workdone in carrying a unit charg from one plate to another.

Thus $V = E_0 (d-t) + E t \longrightarrow (3)$

substitute eqn (1) & (2) in eqn (3)

$$V = \frac{q}{\epsilon_0 A} (d-t) + \frac{q}{K \epsilon_0 A} t$$

$$V = \frac{q}{\epsilon_0 A} \left[(d-t) + \frac{t}{K} \right] = \frac{q}{\epsilon_0 A} \left[d - t \left(1 - \frac{1}{K} \right) \right] \rightarrow (4)$$

The capacitance of the capacitor is given by

$$C = \frac{q}{V} = \frac{q}{\frac{q}{\epsilon_0 A} \left[d - t \left(1 - \frac{1}{K} \right) \right]}$$

$$= \frac{\epsilon_0 A}{\left[d - t \left(1 - \frac{1}{K} \right) \right]}$$

14

$$C = \frac{\epsilon_0 A K}{[Kd - t(K-1)]} \longrightarrow (5)$$

when the dielectric is compleatily filled then $t = d$. In this case the capacitance is given by

$$C = \frac{\epsilon_0 A K}{[Kt - t(K-1)]} = \frac{\epsilon_0 A K}{t} \longrightarrow (6)$$

Effect of electric field on dielectrics

1) Needle shaped cavity :-

consider a dielectric placed in between the parallel charged plates producing electric field 'E' as shown in fig. AB is needle shaped cavity in the dielectric

The length of the cavity is parallel to the direction of electric field E in the dieletric

Let 'C' be the centre of cavity.

Let us consider a unit charge is placed at C. There will be no force On the +ve Charge at C due to the Charges On A (or) B Now the Only force On the Charge at C due to applied Electric field E. Hence the electric field inside a needle shaped cavity in a dielectric is the same as External electric field.

(2) Disc shaped cavity

Consider a disc shaped Cavity AB in the dielectric as shown in fig. the diameter of the Cavity is very large as compared to its thickness. Its Circular Surface is parallel to the plates of a charged condenser Imagine a unit +ve charge at the Center

Due to polarisation of dielectric there will be polarised bound charged on the surfaces of the disc.

These charges are there to the direction of electric field E

The electric field E_0 inside the cavity is given by $E_0 = E + E'$, where E' is the field due to the induced charges on the faces.

By coulomb's law $E' = \frac{\text{Surface density}}{\text{permitivity}} = \frac{\sigma}{\epsilon_0}$

If 'p' be the polarisation, then $p = \sigma$

$\therefore E' = \frac{P}{\epsilon_0}$

Now $E_0 = E + \frac{P}{\epsilon_0} \Rightarrow \epsilon_0 E_0 = \epsilon_0 E + p$

$\therefore D = \epsilon_0 E + p$.

Where electric displacement.

Dielectric strength

Dielectric strength is defined as the electrical strength of an insulating material in a sufficient strong electric field the insulating properties of an insulator breaks down allowing flow of charge.

Dielectric strength is measured as the maximum voltage required to produce a dielectric breakdown through a material. It is expressed as volts per unit thickness for plastic material 1-100Mv/m.

It usually depends on the thickness of the material and on the method and conditions of the test. A high dielectric constant indicates that the material is highly insulating.

The dielectric strength is measured by

Similar methods as the dielectric breakdown voltage but the end point is determined by an increase in conductance in the dielectric under test limiting field which can be sustained. the results are given in v/m.

The dielectric strength of solid electrical insulating materials is determined according to ASTM. standard procedure. this method is a control and acceptance test for direct voltage applications.

The dielectric strength of a material is a measure of its ability to sustain high voltage differences without current breakdown.

— As the voltage across the material is increased at some value of voltage a burst of current transits through the sample and causes severe damage to the material units. volts/cm.

Ex: mineral oil has good dielectric strength & thermal conductive properties. Its insulation level is dependent upon the level of impurities it can be done by oil immersed switch performance. the oil has a co-efficient of expansion of about 0.0008 per °c.

16

Relation b/w D, E & P:

The three electric vector are

1. electric intensity (E)
2. Dielectric polarization (P)
3. Electric displacement (D).

1. electric intensity (E):

The electric intensity (E) at any point in the electric field is numerically equal to the force experienced by unit +ve charge placed at that point.

Dielectric polarisation (P)

the electric Dipole moment per unit volume is called as the dielectric polarision (P).

Electric Displacement (D)

When the Dielectric slab is placed b/w the plate of parallel plate condenser the medium is polarised let 'q'' be the induced surface now the charge on plate be q and these are related as

$$\frac{q}{K\epsilon_0 A} = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A} \rightarrow (i)$$

from (i)

$$\frac{q}{\epsilon_0 A} = \frac{q}{K\epsilon_0 A} + \frac{q'}{\epsilon_0 A}$$

$$\frac{q}{A} = \epsilon_0 \left[\frac{q}{K\epsilon_0 A}\right] + \frac{q'}{A}$$

W.K. $\frac{q}{K\epsilon_0 A} = E$, $\frac{q'}{A} = P$

$$\frac{q}{A} = \epsilon_0 E + P \rightarrow (2)$$

$$D = \epsilon_0 E + P \rightarrow (3)$$

Major - 9.

Electricity

Unit - II current electricity

→ **Electrical conduction :-**

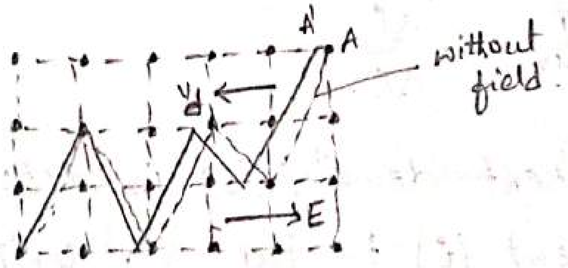
Electrical conduction is the movement of charged particles through a substance, which can result in an electric current.

Electrical conductivity is a function of the structure of the materials and can change by heating.

It depends on the materials.

→ **Drift Velocity :-**

When the current in the conductor is zero and potential difference is maintained across the conductor, the electrons gain some average velocity in the direction of +ve potential. This avg velocity is superimposed over the random velocity and is called as drift velocity as shown in fig.



(2) Let us consider the sample charge distribution of electrons each of charge e and N be the no of electrons per unit volume, then

$$\rho = Ne \quad \text{and} \quad J = NeV_d$$

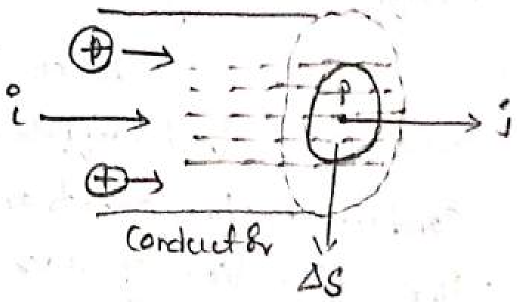
$$V_d = \frac{J}{Ne} \quad \text{and} \quad V_d = \frac{i}{NAe}.$$

where V_d is the drift velocity.

→ **Current Density:-**

If the current is distributed uniformly across a conductor then the magnitude of current density J is equal to current i through infinitesimal area at that point.

$$J = \frac{i}{A} \text{ amp/metre}^2.$$



the area A being normal to the direction of flow of current as shown in fig.

Let Δi be the current through area ΔS then average current density is given by

$$J_{av} = \frac{\Delta i}{\Delta S}.$$

Therefore current density (J) is defined as the ratio of the current (i) to the cross section area (A).

→ Relationship b/w current Density and Drift Velocity :-

Consider fig, let the electric field be maintained between the two ends of a conductor towards the left. The electrons move towards the right. Let V_d be the drift velocity of ~~a~~ the electrons. In a time dt ~~etc~~ each electron advances a distance ~~$t V_d$~~ $l = V_d dt$. The no of electrons crossing the shaded area time dt. If n be no of free electrons per unit volume and e is the charge on each electron then the total charge crossing the shaded area in time dt is.

$$dq = neV_d\,A\,dt \longrightarrow (1)$$

A - area of cross section.

from eq (1), the tot charge crossing a surface in time dt is

$$dq = A\,dt\left[n_1 e v_1 + n_2 e V_2 + \cdots + neV_d\right]$$

and current is $\quad i = \frac{dq}{dt} = AneV_d$

and drift velocity $\quad V_d = \frac{i}{Ane} = \frac{J}{ne} \longrightarrow (2)$

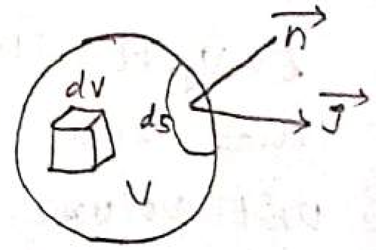
eq (2) represents the relationship b/w current density and drift velocity

(4)

→ Equation of continuity :-

The differential equation relating the current density j and charge density ρ at each point in a circuit is known as equation of continuity. This eq is based on the Law of conservation of charge.

fig shows a small area ds on the surface of an irregular volume V. Let n be the unit vector ⊥er to ds and j be current density through ds. The current through ds in a direction ⊥er to it is given by $= j.n.ds.$



∴ current through the whole surface

$$i = \iint_S j.n.ds \longrightarrow (1)$$

Consider a small elementary volume dv inside the surface. Let ρ be the charge density i.e charge per unit volume. Then the charge in volume dv will be ρdv.

∴ charge in the whole volume $= \iiint_V \rho\, dv$

So the rate of decrease of charge

$$= -\frac{\partial}{\partial t}\iiint_V \rho\, dv \longrightarrow (2)$$

According to the Law of conservation of charge, eq (1) should be equal to eq (2). Hence

$$\iint_S j \cdot n \cdot ds = -\frac{\partial}{\partial t}\iiint_V \rho \, dv \longrightarrow (3)$$

According to Gauss divergence theorem

$$\iint_S j \cdot n \cdot ds = \iiint_V \nabla \cdot j \cdot dv$$

$$\therefore \iiint_V \nabla \cdot j \cdot dv = -\frac{\partial}{\partial t}\iiint_V \rho \, dv.$$

$$\iiint_V \nabla \cdot j \cdot dv = \iiint_V \left(-\frac{\partial \rho}{\partial t}\right) dv$$

$$\iiint_V \left(\nabla \cdot j + \frac{\partial \rho}{\partial t}\right) dv = 0 \longrightarrow (4)$$

This integral must be zero for any arbitrary volume. This is possible when the integral is zero. Thus

$$\nabla \cdot j + \frac{\partial \rho}{\partial t} = 0. \longrightarrow (5)$$

This is known as equation of continuity.

For steady current $\frac{\partial \rho}{\partial t} = 0$. So in this case

$$\left.\begin{aligned} \nabla \cdot j &= 0 \\ \text{div } j &= 0. \end{aligned}\right\} \longrightarrow (6)$$

⑥ Ohm's Law and Limitations:-

According to Ohm's Law, if the physical conditions such as temp remain constant, the current between two points in a conductor is proportional to the potential difference between these two points,

i.e $i \propto V$ & V/i = a const (R).

Where R is known as resistance of the conductor.

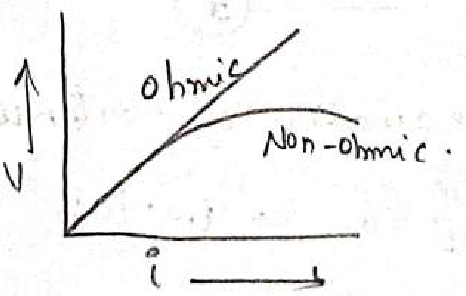
SI units R is in ohm

if I is in ampere, V is in volts.

Hence

1) The conductors which obey ohm's law are called ohmic conductors. (Eg:- metals & alloys).

the graph b/w current and potential diff is a straight line passing through origin.



2) The conductors which do not obey ohm's law are known as non-ohmic conductors. as shown in fig. (Eg:- Diode valve, neon gas, junction diode)

Limitations:-

For a resistor obeys ohm's law, the graph of current as a function of potential difference is a straight line as shown in fig ⓐ. When sign of PD change, then sign of current also change. fig (a).

(7)

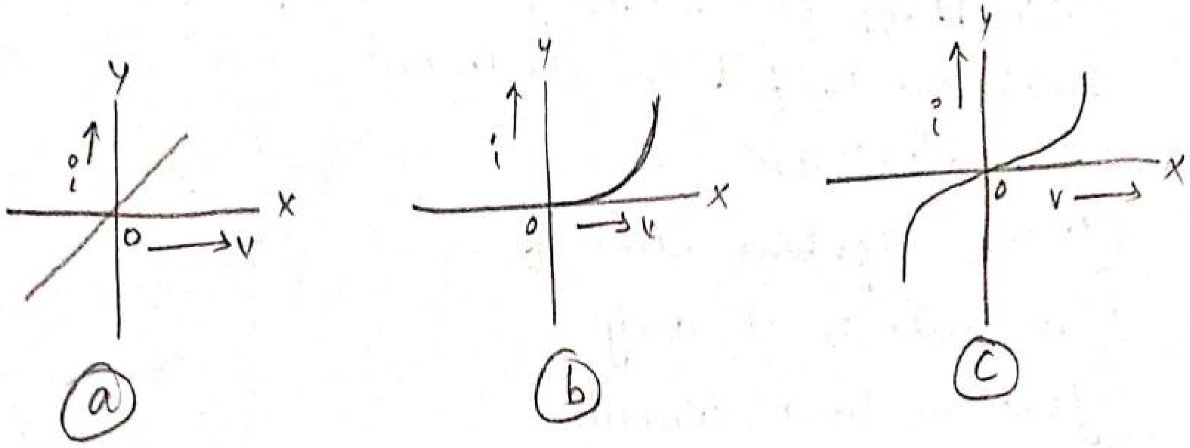


fig (b) for vacuum diode. for +ve potentials, the cathode current is proportional to $V^{3/2}$. for -ve potentials of anode, current is small, so the device does not obey ohm's law.

fig (c) shows v-i characteristics of p-N junction semi conductor. When p-N junction is forward biased, +ve terminal to p, -ve terminal to N, then variation in current with PD is Very Rapid.

While p-N junction is Reversed biased -ve terminal to p side and +ve terminal to N side the variation of current is very Small and in ~~posi opposite~~ opposite direction

---

ohm's law is unable to give current in complicated circuits. Kirchhoff in 1842, gave two general laws which are extremely useful in electrical circuits. ⟶

(8) → Kirchhoff's Law :—

Kirchhoff in 1842, gave two general laws which are useful in electrical circuits. These are

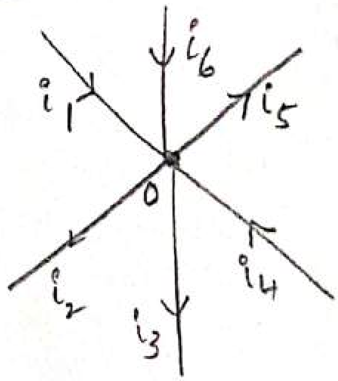


fig (1)

1, 6, 4 – in
2, 3, 5 – out.

(1) The algebric sum of the currents at any junction in a circuit is zero. i.e.

$$\boxed{\Sigma i = 0} \rightarrow (1)$$

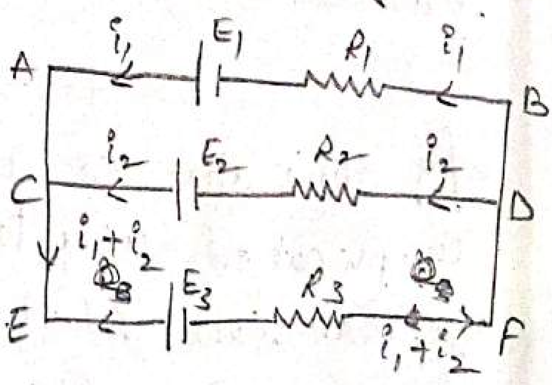
This means the current which flow towards a point are taken as +ve while those ~~of~~ flow away from the point are taken as –ve. From fig. we have

$$i_1 - i_2 - i_3 + i_4 - i_5 + i_6 = 0.$$

(2) In any closed circuit, the algebric sum of the products of the current and resistance of each part of the circuit is equal to the total emf in the circuit. i.e

$$\boxed{\Sigma iR = \Sigma E} \rightarrow (2)$$

fig (2)



The product of current and resistance is taken as +ve when we thraustvese ~~the~~ in the direction of current. The emf is taken as +ve when we traverse from –ve to +ve electrode.

through electrolyte. (9)

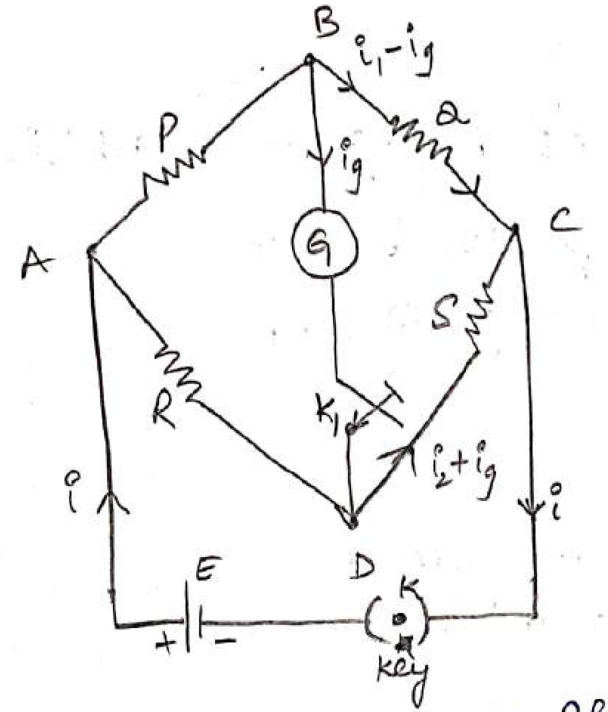
Let us apply Kirchhoff's ~~Law~~ Second Law for fig (2)

for the mesh ACDBA = $i_1R_1 - i_2R_2 = E_1 - E_2$ ⟶ (3)

for the mesh EFDCE = $i_2R_2 + (i_1+i_2)R_3 = E_2$. ⟶ (4)

for the mesh EFBAE = $i_1R_1 + (i_1+i_2)R_3 = E_1$ ⟶ (5)

→ Wheatstone bridge - balancing condition :-



The Wheatstone's bridge is shown in fig. To consider the condition of balance in this bridge we apply Kirchhoff's law to different meshes.

Applying Kirchhoff's 2nd law to mesh ABDA, we have

$$i_1P + i_gG - i_2R = 0 . \longrightarrow (1)$$

for the mesh BCDB, we have

$$(i_1 - i_g)Q - (i_2 + i_g)S - i_gG = 0 \longrightarrow (2)$$

(10) When the bridge is balanced, no current flows through galvanometer. G. i.e $i_g = 0$.
from eq (1) and eq (2) we have

$$i_1 P = i_2 R \longrightarrow (3)$$

$$i_1 Q = i_2 S \longrightarrow (4)$$

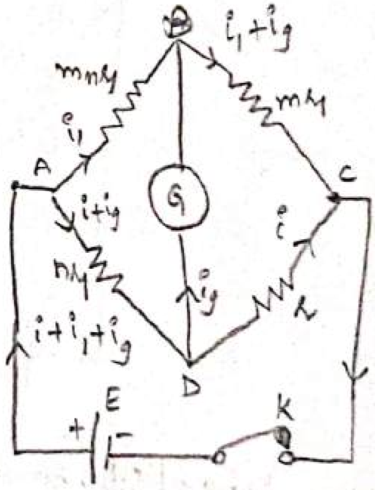
from (3) & (4) $\dfrac{P}{Q} = \dfrac{R}{S}$

This is the condition for the balance of Wheatstone's bridge.

→ **Wheatstone's bridge - Sensitivity :-**

The condition for the balance of Wheatstone bridge is given by $\dfrac{P}{Q} = \dfrac{R}{S}$

When there is no current in the galvanometer



Now Consider that bridge is slightly out of balance, that means there will be slightly current in the galvanometer. So the Sensitivity of the bridge depends on the sensitivity of the galvanometer. The Sensitivity of Wheatstone's bridge calculation arrangement is shown in fig.

(11)

Let r be the unknown resistance which is connected in the fourth arm of Wheatstone's bridge and i be the current in it. Let the resistance in the arms AB, BC, AD be mnr_1, mr_1 and nr_1 respectively. Let the current in the galvanometer be i_g. The current distribution in other arms is shown in fig.

When the bridge is balanced $i_g = 0$.

and $$\frac{mnr_1}{mr_1} = \frac{nr_1}{r} \quad \& \quad r = r_1$$

Therefore the difference $(r - r_1)$ is the measure of the want of balance i.e. the bridge will be most sensitive if the deflection in the galvanometer is large even when $(r - r_1)$ is extremely small.

The current through the galvanometer for a given want of balance can be calculated by applying the Kirchhoff's law to the meshes ABDA and BCDB respectively.

$$mnr_1 i_1 + Gi_g - nr_1(i + i_g) = 0 \longrightarrow (1)$$

and $$mr_1(i_1 + i_g) - ri + Gi_g = 0 \longrightarrow (2)$$

These equations may be written as

~~$mnr_1 i_1 +$~~

$$mnr_1 i_1 - Gi_g - nr_1 i - nr_1 i_g = 0 \longrightarrow (3)$$

$$mr_1 i_1 + mr_1 i_g - ri + Gi_g = 0 \longrightarrow (4)$$

(12) Multiply eq (4) by n

$$mnr_1 i_1 + mnr_1 i_g - nri + nGi_g = 0 \quad \longrightarrow (5)$$

Substracting eq (5) from (4) we get (5) − (4)

$$-Gi_g - nr_1 i - nr_1 i_g - mnr_1 i_g + nri - nGi_g = 0 \quad \longrightarrow (6)$$

$$-(1+n)G i_g + ni(r - r_1) - (1+m)n r_1 i_g = 0$$

Dividing by ni, we get

$$-\frac{(1+n)}{n} G \frac{i_g}{i} + (r - r_1) - (1+m) r_1 \cdot \frac{i_g}{i} = 0.$$

$$\frac{i_g}{i}\left(\frac{1+n}{n}\right)G + (1+m) r_1 \cdot \frac{i_g}{i} = (r - r_1)$$

$$\frac{i_g}{i}\left[\left(\frac{1+n}{n}\right)G + (1+m) r_1\right] = (r - r_1)$$

$$\frac{i_g}{i} = \frac{(r - r_1)}{\left[\left(\frac{1+n}{n}\right)G + (1+m) r_1\right]} \quad \longrightarrow (7)$$

This expression gives the current i_g in the galvanometer for a given want of balance $(r - r_1)$. Thus is independent of the resistance of the battery. The ratio i_g/i depends on the want of balance $(r - r_1)$, The galvanometer resistance G and the

the ratio of m and n

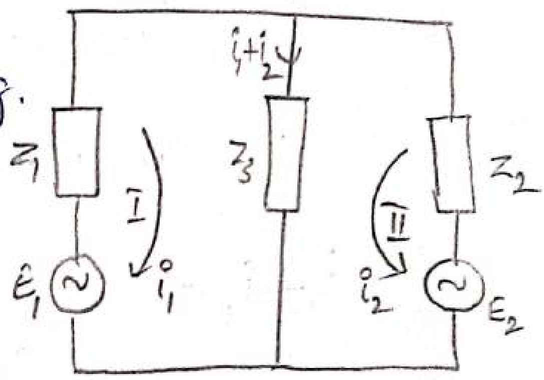
for greater sensitiveness for a given want of balance $(h-h_1)$, the value of i_g should be large. Therefore,

1) i should be large
2) G should be small
3) m should be small.
4) n should be large.

→ **Superposition Theorem:-**

Statement:- In any linear network containing linear impedances and energy sources the current in any element is the vector sum of the currents that are separately caused in that element if each source of emf were considered separately of course, all other sources being replaced at that time by their internal impedences.

fig ①



Consider a network shown in fig. The ckt has two emf sources E_1 and E_2 having internal impedences Z_1 and Z_2. Let we have to find out the current through an impedence Z_3. This can be obtained by superposition theorem in the following way.

1) Imagine that emf source E_2 is removed ~~by~~ but

not its internal impedance Z_2. Let i_1 be the current in Z_3 due to E_1 alone.

2) Imagine that emf source E_1 is removed but not its internal impedence Z_1. Let i_2 be the current in Z_3 due to E_2 alone.

3) The sum $(i_1 + i_2)$ is the current through Z when both emf source are operating.

Proof :- Applying Kirchhoff's Voltage Law to the two meshes I & II of fig ①, we have

$$E_1 = (Z_1 + Z_2) I_1 + Z_3 I_2 \longrightarrow ①$$

and $$E_2 = Z_3 I_1 + (Z_2 + Z_3) I_2 \longrightarrow ②$$

Solving these eq's for I_1 and I_2 we get

$$I_1 = \frac{(Z_2+Z_3)E_1}{Z_1Z_2+Z_2Z_3+Z_3Z_1} - \frac{Z_3E_2}{Z_1Z_2+Z_2Z_3+Z_3Z_1} \longrightarrow ③$$

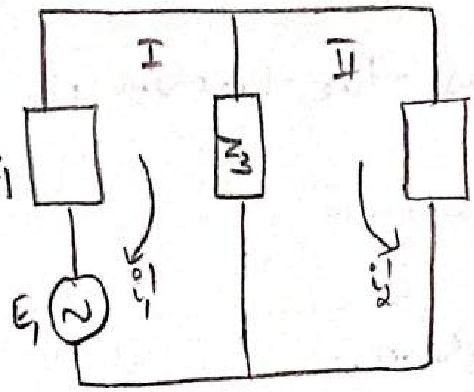
and

$$i_2 = \frac{(Z_1+Z_2)E_2}{Z_1Z_2+Z_2Z_3+Z_3Z_1} - \frac{Z_3E_1}{Z_1Z_2+Z_2Z_3+Z_3Z_1} \longrightarrow ④$$

∴ The current through Z_3 is given by

$$i_1 + i_2 = \frac{Z_2E_1 + Z_1E_2}{Z_1Z_2+Z_2Z_3+Z_3Z_1} \longrightarrow ⑤$$

When E_2 is removed, the network is shown in fig ②. Let i_1' and i_2' be the currents in two meshes respectively. Applying Kirchhoff's Law, we have fig 2



$$E_1 = (Z_1+Z_3)\,i_1' + Z_3 i_2' \longrightarrow (6)$$

$$0 = Z_3 i_1' + (Z_2+Z_3)\,i_2' \longrightarrow (7)$$

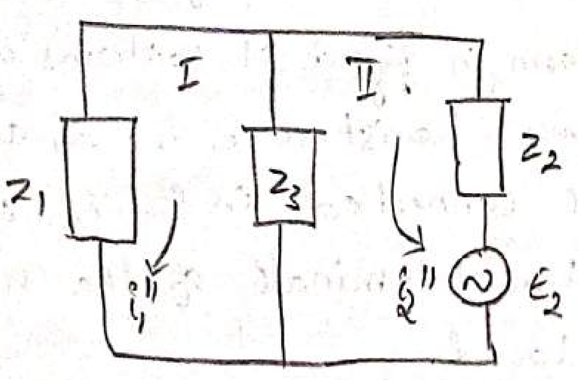
Solving eq's (6) & (7) we get

$$i_1' = \frac{(Z_2+Z_3)E_1}{Z_1Z_2+Z_2Z_3+Z_3Z_1} \longrightarrow (8)$$

and $$i_2' = -\frac{Z_3E_1}{Z_1Z_2+Z_2Z_3+Z_3Z_1} \longrightarrow (9)$$

When E_1 is removed, the network is shown in fig (3). Let i_1'' and i_2'' be the currents in two meshes. Applying Kirchhoff's Law we have.

fig 3



$$0 = (Z_1+Z_3)\,i_1'' + Z_2 i_2'' \longrightarrow (10)$$

$$E_2 = Z_3 i_1'' + (Z_2+Z_3)\,i_2'' \longrightarrow (11)$$

Solving eq's we have

$$i_1'' = -\frac{Z_3E_2}{Z_1Z_2+Z_2Z_3+Z_3Z_1} \longrightarrow (12)$$

and $$i_2'' = \frac{(Z_1+Z_3)E_2}{Z_1Z_2+Z_2Z_3+Z_3Z_1} \longrightarrow (13)$$

from these eq's

it is real that

(16) $i_1 = i_1' + i_1''$

and $i_2 = i_2' + i_2''$ Thus the theorem is proved.

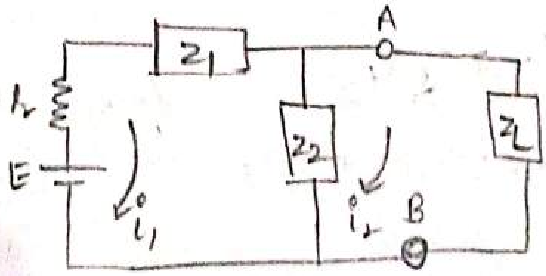
→ Thevenin's Theorem :-

Statement :- Any two terminals of a network, containing sources of impedence and linear impedences, can be replaced by an equilent circuit consisting of a voltage source E_{th} in series with impedence Z_{th}.

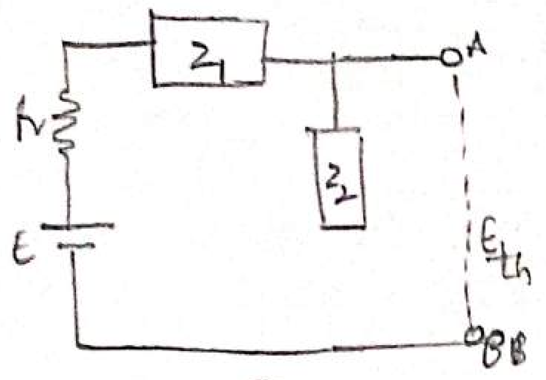
The emf of the voltage source is equal to the P.D b/w the two terminals in open cket position. The series impedence is equal to the impedence b/w two terminals when all sources being replaced by their internal impedence.

Procedure :- The procedure for drawing the Thevenin's equilant circuit is as follows :-

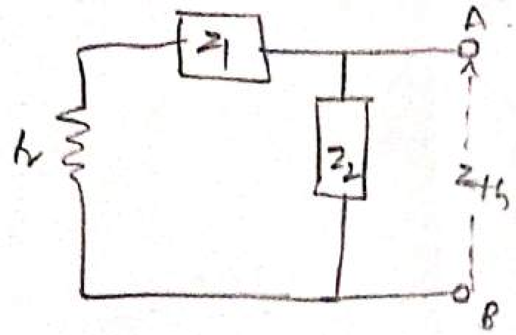
Consider a network shown in fig ⓐ It contains a d.c source of emf E of internal resistance r. Z_1 and Z_2 are the two impedences connected in series with source. A and B are two terminals of the network and Z_L is the external load.



ⓐ



(b)



(c)

First, with load terminals A and B open fig (b), we calculate the open circuited emf of voltage source E_{th}. for this purpose we calculate the current flowing in the circuit of fig (b). This is given by

$$i = \frac{E}{r + Z_1 + Z_2}.$$

Now voltage across Z_2 is given by

$$E_{th} = \text{current} \times \text{impedence} = \left(\frac{E}{r + Z_1 + Z_2}\right) Z_2 \rightarrow (1)$$

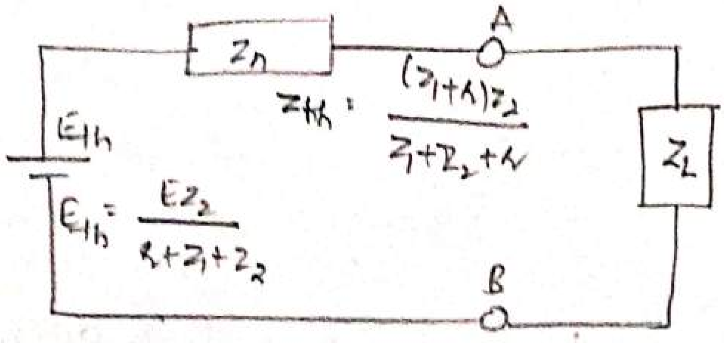
The next step is to calculate the Z_{th}. The ckt for calculation Z_{th} is shown in fig (c). Here battery is removed while its internal impedence r is considered.

corresponding to points A and B, Z_1 and r are in series and equivalent impedence of the two is in parallel with Z_2, Hence

$$Z_{eq} = (Z_1 + r)$$

$$\text{Now } \frac{1}{Z_{th}} = \frac{1}{Z_2} + \frac{1}{(Z_1 + r)} = \frac{(Z_1 + r) Z_2}{(Z_1 + Z_2 + r)} \rightarrow (2)$$

(18) The Thevenin's equivalent ckt is shown in fig (d)



<u>Proof:-</u> Let i_1 and i_2 be the currents into meshes as shown in fig (a) Applying ~~the~~ Kirchhoffs law, we have

$$(h+Z_1+Z_2)i_1 - Z_2 i_2 = E \longrightarrow (3)$$

and $$(Z_2+Z_L)i_2 - Z_2 i_1 = 0 \longrightarrow (4)$$

from eq (4) $$i_1 = \left(\frac{Z_2+Z_L}{Z_2}\right) i_2 .$$

Substituting this value of i_1 in eq (3), we get

$$\cancel{(h+Z_1+Z_2)\left(i\left(\frac{Z_2+Z_L}{Z_2}\right)\right)}$$

$$(h+Z_1+Z_2)\left(\frac{Z_2+Z_L}{Z_2}\right)i_2 - Z_2 i_2 = E$$

$$i_2\left[\frac{(h+Z_1+Z_2)(Z_2+Z_L)}{Z_2} - Z_2\right] = E$$

$$i_2 = \frac{E}{\left[\frac{(h+Z_1+Z_2)(Z_2+Z_L)}{Z_2} - Z_2\right]}$$

$$= \frac{E\left[\dfrac{Z_2}{(r+Z_1+Z_2)}\right]}{\left[(Z_2+Z_L)-\left(\dfrac{Z_2^2}{r+Z_1+Z_2}\right)\right]}$$

$$= \frac{E\left[\dfrac{Z_2}{r_1+Z_1+Z_2}\right]}{Z_2-\dfrac{Z_2^2}{(r+Z_1+Z_2)}+Z_L.}$$

$$= \frac{E\left[\dfrac{Z_2}{r+Z_1+Z_2}\right].}{Z_2\left[1-\dfrac{Z_2}{r+Z_1+Z_2}\right]+Z_L}$$

$$\therefore i_2 = \frac{E\left[\dfrac{Z_2}{r_2+Z_1+Z_2}\right]}{Z_2\left[\dfrac{Z_1+r}{r+Z_1+Z_2}\right]+Z_L.} \longrightarrow (5)$$

From eq (5) that the term $E\left[\dfrac{Z_2}{r+Z_1+Z_2}\right]$ is the open circuited voltage across the terminals A and B while the term $Z_2\left[\dfrac{Z_1+r}{r+Z_1+Z_2}\right]$ is the impedence of the resulting network between terminals A and B when emf source is short circuited. Hence from eq (1) & (2) theorem is proved.

20

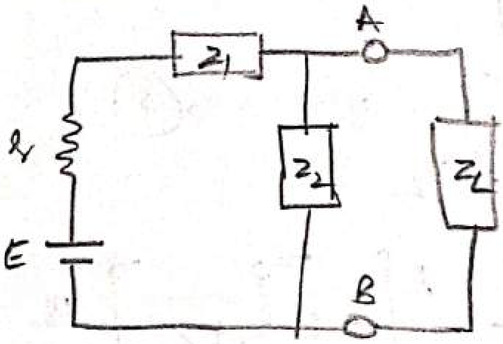
Norton's Theorem :-

Statement :- Any two terminal linear network containing sources and impedences can be replaced by a constant current source in parallel, with a single impedance.

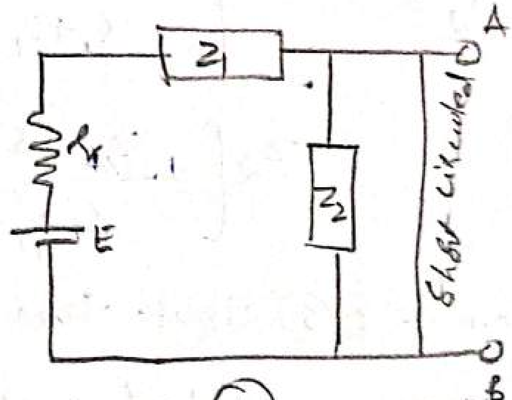
The current is the short circuited current between the two terminals of the network while the impedence between the two terminals is the impedence when all sources of emf are replaced by their internal impedance.

Procedure :- The procedure for drawing the Norton's equivalent is as follows.

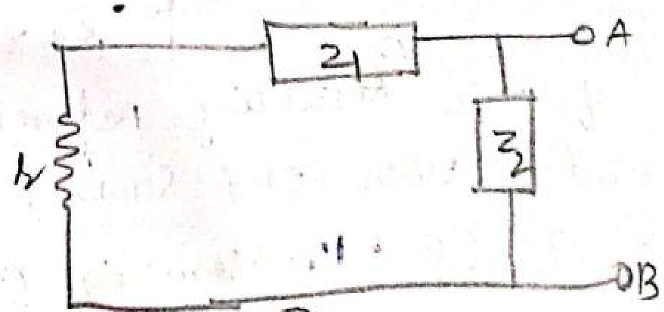
Consider a network shown in fig a. Let us draw the Norton's ~~equilent~~ equivalent of this current.



(a)



(b)



(c)

first the terminals A and B are short circuited fig (b) Now Z_2 becomes ineffective. The current flowing in the circuit is given by $\frac{E}{r+Z_1}$
This is denoted by i_N. ~~The~~ Hence

$$i_N = \frac{E}{r+Z_1} \longrightarrow (1)$$

The next step is to open the terminals A and B Now battery E is removed. its internal impedence is considered as shown in fig (c). The " Z_n of the ckt as looked into from open terminals A and B is given by

$$Z_N = \frac{(Z_1+r)Z_2}{(Z_1+r)+Z_2} \longrightarrow (2)$$

because Z_1 and r are in series and then equivalent impedence is in parallel with Z_2.
Thus the Norton's equivalent circuit is shown in fig (d)

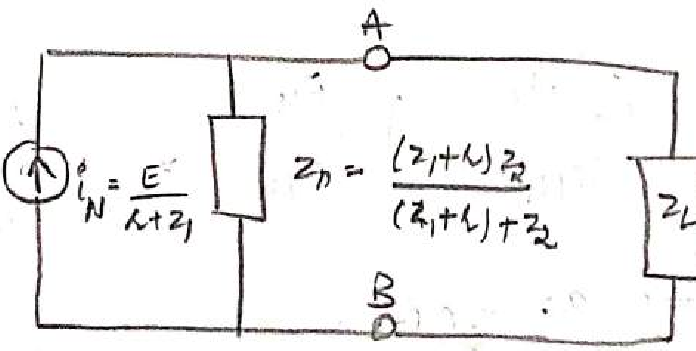
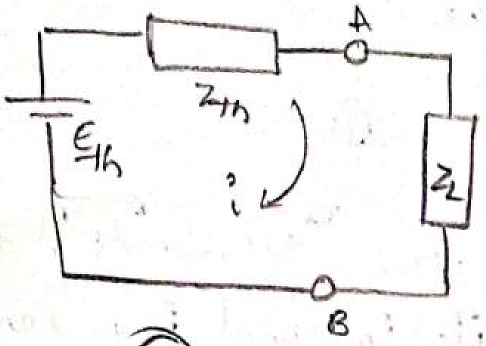


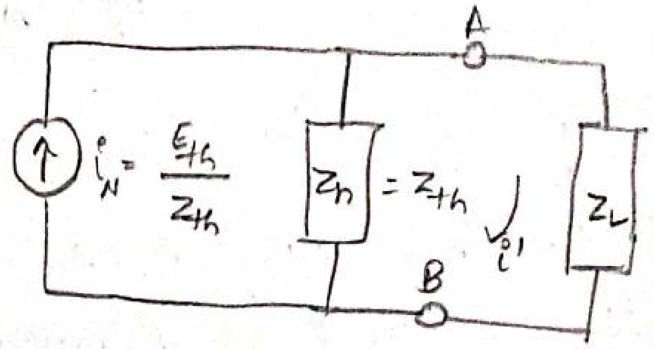
fig (d)

<u>Proof</u>:- The Norton's equivalent circuit may be found from Thevenin equivalent ckt.

(22) Consider a Thevenin ckt shown in fig (e). Its Norton's equivalent ckt is shown in fig (f).



(e)



We shall show that both the circuits are equivalent. The current through the load Z_L in Thevenin ckt is given by

$$i = \frac{E_{th}}{Z_{th} + Z_L} \longrightarrow (3)$$

The current flowing through load in Norton's circuit is given by

$$i' = \left(\frac{Z_{th}}{Z_{th} + Z_L}\right) i_N \quad \text{(by current division law.)}$$

$$= \left(\frac{Z_{th}}{Z_{th} + Z_L}\right) \frac{E_{th}}{Z_{th}} = \frac{E_{th}}{Z_{th} + Z_L} \longrightarrow (4)$$

eq (4) is same as eq (3).

This proves the Norton's theorem.

UNIT - IV

1. Electromagnetic waves and maxwell's Equations

Idea of displacement current :-

A changing electric field is equalent to current which flows along the electric field is changing and produces magnetic effect. this is known as displacement current.

* Amphere's law is vector form can be defined as.

$$\nabla \times B = \mu_0 j \longrightarrow ①$$

where j = current density

* Taking divergents of this equation

$$\nabla \cdot (\nabla \times B) = \text{div curl} B = \text{div} \mu_0 j = \mu_0 \text{div} j \longrightarrow ②$$

* The divergents of a curl of a vector is always zero

$$\text{div} j = 0$$

This shows that the total flux of a current out of any closed surface is zero, then eqn ② is contradiction of eqn of continuity.

$$\text{div} j + \frac{\partial \rho}{\partial t} = 0 \longrightarrow ③$$

where ρ represents charge density.

Maxwell and amphere conclude that eq ① as

$$\text{curl} B = \mu_0 j + \text{something} \longrightarrow ④$$

Here something is known as changing electric field is equallent to current which flows along ay, and produce some magnetic field is known as displacement current.

In vector form the Gauss Law expressed

$$\nabla D = \rho$$

$$\nabla \frac{\partial D}{\partial t} = \frac{\partial \rho}{\partial t}$$

1

Adding $\nabla \cdot J$ on both sides we get

$$\nabla \cdot j + \frac{\partial \rho}{\partial t} = \nabla j + \nabla \frac{\partial D}{\partial t}$$

$$= \nabla \left(j + \frac{\partial D}{\partial t} \right)$$

Thus, $\nabla j = 0$ for steady current.

$$\nabla \left(j + \frac{\partial D}{\partial t} \right) = 0 \text{ energy where} \longrightarrow (5)$$

For steady current as well as current corres-ponding to time where. In this way maxwell's replace into 'j' in ampere law by

$$\left(j + \frac{\partial D}{\partial t} \right) \longrightarrow (6)$$

Thus ampere's law becomes eqn (6)

The term $\frac{\partial D}{\partial t}$ is called displacement current density because it arises when electric displacement 'd' changes with time eqn (6) changes and return as eqn (7)

$$\text{curl } B = \mu_0 \left[j + \epsilon_0 \frac{\partial E}{\partial t} \right] \longrightarrow (7)$$

Maxwell's in 1862 formulated the basic law of electricity and magnetism in four fundamental eqns and are known as maxwell's eqns.

The integral form of these eqn are given below:

$$\oint E \cdot ds = \frac{q}{\epsilon_0} \longrightarrow (1)$$

$$\oint B \cdot ds = 0 \longrightarrow (2)$$

$$\oint E \cdot dl = \frac{-d\phi_B}{dt} \longrightarrow (3)$$

$$\oint B \cdot dl = \mu_0 \left[j + \epsilon_0 \frac{\partial E}{\partial t} \right] \longrightarrow (4)$$

Maxwell Eqns can also be stated in the differential form as followed.

$$\text{div } E = \frac{\rho}{\epsilon_0} \longrightarrow (5)$$

$$\text{div } B = 0 \longrightarrow (6)$$

$$\text{curl } E = \frac{-\partial B}{\partial t} \longrightarrow (7)$$

$$\text{curl } B = \mu_0 [J + \epsilon_0 \frac{\partial E}{\partial t}] \longrightarrow (8)$$

3

The above diff forms can be obtained from the integral forms as followed.

(1) $\oint E\,ds = \frac{q}{\epsilon_0}$

If ρ be the charge density and dv, the small volume considered, then

$$\rho = \int_V \rho dv$$

$$\therefore \oint E \cdot ds = \frac{1}{\epsilon_0} \int_V \rho dv \quad \text{or} \quad \oint \epsilon_0 E \cdot ds = \int_V \rho dv$$

$$\text{i.e} \quad \oint D \cdot ds = \int_V \rho dv \quad (\because \epsilon_0 E = D)$$

$$\int_V \nabla \cdot D dv = \int_V \rho dv \quad \text{(by divergence theorem)}$$

$$\nabla \cdot D = \rho$$

$$\text{or} \quad \nabla \cdot E = \frac{\rho}{\epsilon_0} \quad \text{or curl } E = \frac{\rho}{\epsilon_0}$$

$$\text{or} \quad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\epsilon_0}$$

(2) $\oint B \cdot ds = 0$

Transforming the surface integral into volume integral we get

$$\int_V \nabla B dv = 0$$

As the volume is orbitary. the integral must be zero. $\nabla \cdot B = 0$ (or)

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

4

3. $\oint E \cdot dl = \frac{-d\phi_B}{dt}$

$= \frac{-\partial}{\partial t} \int_s B \cdot ds$

$= -\int_s \frac{\partial B}{\partial t} ds$

Applying stoke's theorem

$-\oint E \cdot dl = \int_s (\nabla \times E) ds$

$\therefore \int_s (\nabla \times E) ds = -\int_s \frac{\partial B}{\partial t} ds$

As the equation is true for all surfaces, we have

$\nabla \times E = \frac{-\partial B}{\partial t}$ (or) curl $E = \frac{-\partial B}{\partial t}$

(or) $i\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right] + j\left[\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right] + k\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right]$

$= \frac{-\partial}{\partial t}\left[iB_x + jB_y + kB_z\right]$

$\therefore \left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right] = \frac{-\partial B_x}{\partial t}$

$\left[\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right] = \frac{-\partial B_y}{\partial t}$

$\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right] = \frac{-\partial B_z}{\partial t}$

(4) $\oint B \cdot dl = \mu_0 i$

using stoke's theorem

$\oint B \cdot dl = \int_s (\nabla \times B) ds$

$\int_s (\nabla \times B) ds = \mu_0 \int_s j \cdot ds$

or $\nabla \times B = \mu_0 j$

or $\nabla \times B = \mu_0 \left(j + \varepsilon_0 \frac{dE}{dt}\right)$

Replacing j by $\left(j + \varepsilon_0 \frac{\partial E}{\partial t}\right)$,,

Maxwell's wave Equation of electromagnetic waves :-

Let us apply maxwell's electromagnetic Eqn to Homogeneous hysophopic dielectric medium. In dielectric medium conductivity is $J=0$ and charged density $\rho=0$ and also

charged density $\rho=0$, $D=KE_0$

$E=CE$

$H=\epsilon H$

Hence maxwell eqns for Dielectric become

maxwell's Eqn dielectric medium $B=\mu_0\mu_r H$

$$\nabla\cdot B=0 \longrightarrow ①$$

$$\nabla\cdot E=0 \longrightarrow ②$$

$$\nabla\times E=\frac{-\partial B}{\partial t} \longrightarrow ③$$

$$\nabla\times B=\mu\epsilon\frac{\partial E}{\partial t} \longrightarrow ④$$

Taking used Eq ④ we get

$$\nabla\times(\nabla\times B)=\nabla\times\left(\mu\epsilon\frac{\partial E}{\partial t}\right)$$

$$=\mu\epsilon\left(\nabla\times\frac{\partial E}{\partial t}\right)$$

$$=\mu\epsilon\frac{\partial}{\partial t}(\nabla\times E)$$

$$\nabla\times(\nabla\times B)=\mu\epsilon\frac{\partial}{\partial t}\left[\frac{-\partial B}{\partial t}\right] \text{ from Eqn ③}$$

$$\nabla\times(\nabla\times B)=-\mu\epsilon\frac{\partial^2 B}{\partial t^2} \longrightarrow ⑤$$

we know that

$$\nabla\times(\nabla\times B)=\nabla\cdot(\nabla\cdot B)-\nabla^2 B$$

from Eqn ②

$$=\nabla(0)-\nabla^2 B$$

$$\nabla\times(\nabla\times B)=-\nabla^2 B \longrightarrow ⑥$$

comparing eq ⑤ & ⑥

$$-\nabla^2 B=-\mu\epsilon\frac{\partial^2 B}{\partial t^2}$$

$$\nabla^2 B=\mu\epsilon\frac{\partial^2 B}{\partial t^2} \longrightarrow ⑦$$

Taking curl eqn ③

$$\nabla \times (\nabla \times E) = \nabla \times \left(\frac{-\partial B}{\partial t}\right)$$

$$= \frac{-\partial}{\partial t}(\nabla \times B) \text{ from Eqn ④}$$

$$\nabla \times (\nabla \times E) = \frac{-\partial}{\partial t}\left(\mu\epsilon \frac{\partial E}{\partial t}\right)$$

$$\nabla \times (\nabla \times E) = -\mu\epsilon \frac{\partial^2 E}{\partial t^2} \longrightarrow ⑧$$

6

we know that

$$\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 E$$

from equation ①

$$\nabla \times (\nabla \times E) = \nabla(0) - \nabla^2 E \longrightarrow ⑨$$

comparing ⑧ & ⑨

$$-\nabla^2 E = -\mu\epsilon \left(\frac{\partial^2 E}{\partial t^2}\right)$$

$$\nabla^2 E = \mu\epsilon \cdot \left(\frac{\partial^2 E}{\partial t^2}\right) \longrightarrow ⑩$$

Eq ⑦ and ⑩ indicates wave propagation in three dimensional space. these waves involved periodic variations of electric and magnetic field electromagnetic waves. so they are called electromagnetic waves.

Transverse wave nature of electromagnetic waves:-

Consider the case of electromagnetic wave is which the components of vector E and B vary with one co-ordinate say 'x' and also with time 't'. That is $E = E(x,t)$ and $B = B(x,t)$

But $\nabla \cdot E = 0$ (or) $\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$

$$\frac{\partial E_x}{\partial x} = 0 \quad \text{or} \quad E_x = \text{cost} \longrightarrow ①$$

But $\nabla \cdot B = 0$ (or) $\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$

$$\frac{\partial B_x}{\partial x} = 0 \Rightarrow B_x = \text{cost} \longrightarrow ②$$

Eqn ① & ② are obtained on the fact that the derivative of E and B with respect to y and z are zero.

Further $\nabla \times E = \text{curl } E = \frac{-\partial B}{\partial t} \longrightarrow ③$

7

$$\text{curl} = \frac{-\partial B}{\partial t}$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \frac{-\partial}{\partial t}\left[iB_x + jB_y + kB_z\right]$$

$\text{curl } B = \frac{-\partial E}{\partial t}$

$= E_z = \text{const}$

$$i\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right] = \frac{-i\partial B_x}{\partial t}$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = \frac{-\partial B_x}{\partial t} = 0 \longrightarrow ④$$

$$\frac{\partial E_z}{\partial y} = \frac{\partial E_y}{\partial z} = 0$$

from Eqn ④ $\frac{\partial B_x}{\partial t} = 0$, $B_{y}x$ = constant $\longrightarrow ⑤$

similarly taking curl B we show that x = constant

Hence we conclude that e_x and v_x are constants with time and space thus

$$E = jE_y + kE_z \quad \text{and}$$

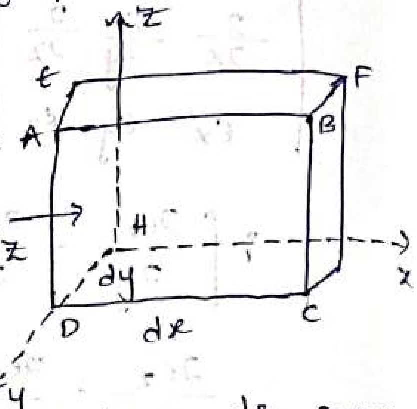
$$B = jB_y + kB_z$$

As vectors E and B do not contain any x component. Hence x direction is the propagation of wave both these vectors are perpendicular to the direction of propagation. Hence maxwell electromagnetic waves are purely transverse in nature.

Poynting Theorem :-

The amount of field Energy passing through unit area of the surface ⊥er to the direction of propagation of Energy is called poynting vector. This is denoted by 'p'. In the plane electromagnetic wave E and B are ⊥er to Each other. And also to the direction of propagation.

Consider an elementary value in the form of rectangular parallelo pipe of sides dx, dy and dz as shown in figure.



The volume of the parallelo pipe is dx, dy, dz suppose the electromagnetic energy is propagating along x-axis, now the area is ⊥er to the direction of propagation of Energy dy, dz

Let electromagnetic Energy is u in the volume then the rate of Energy is $\frac{du}{dt}$ then $\frac{du}{dt} = -\oint p\,ds$.

Negative sign indicates the Energy is entering in the volume.

$$\oint p\,ds = \frac{-\partial u}{\partial t} \rightarrow (1)$$

Then we know that

1. The Energy density per unit volume in electric field is given by $u_E = \frac{1}{2}\epsilon_0 E^2$
2. The Energy density per unit volume in magnetic field is given by $u_B = \frac{1}{2}\mu_0 H^2$

Total Energy $u = \left(\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\mu_0 H^2\right)$

The rate of decrease of Energy for volume dv is equal to is given by

$$-\frac{\partial u}{\partial t} = \frac{-\partial}{\partial t}\left[\tfrac{1}{2}\epsilon_0 E^2 + \tfrac{1}{2}\mu_0 H^2\right] d\cancel{x}$$

Q9

$$-\frac{\partial u}{\partial t} = \frac{-\partial}{\partial t}\left(\tfrac{1}{2}\epsilon_0 E^2 + \tfrac{1}{2}\mu_0 H^2\right) dv$$

$$= \int_V -\left[\epsilon_0 E\left(\frac{\partial E}{\partial t}\right) + \mu_0 H\left(\frac{\partial H}{\partial t}\right)\right] dv \longrightarrow (2)$$

From maxwell's equation, we know that

$$\nabla \times H = j\frac{dD}{dt} = \frac{\partial D}{\partial t} = \epsilon_0 \frac{\partial E}{\partial t} \quad (D = \epsilon_0 E)$$

$$\nabla \times H = \epsilon_0 \frac{\partial E}{\partial t}$$

$$\frac{\partial E}{\partial t} = \frac{\nabla \times H}{\epsilon_0} \longrightarrow (3)$$

$$\nabla \times E = \frac{-\partial B}{\partial t} = -\mu_0 \frac{\partial H}{\partial t}$$

$$\frac{\partial H}{\partial t} = \frac{-\nabla \times E}{\mu_0} \longrightarrow (4)$$

substitute in (3) (4) in eqn (2)

$$-\frac{\partial u}{\partial t} = \int_V -\left[\epsilon_0 E\left(\frac{\nabla \times H}{\epsilon_0}\right) - \mu_0 H\left(\frac{\nabla \times E}{\mu_0}\right)\right]$$

$$-\frac{\partial u}{\partial t} = \int_V \left[H\cdot(\nabla \times E) - E(\nabla \times H)\right] dv$$

$$\{\nabla\cdot(A\times B) = B(\nabla \times A) - A(\nabla \times B)\}$$

$$-\frac{\partial u}{\partial t} = \int_V \nabla\cdot(E\times H)\, dv$$

using Gauss theorem of divergents the volume integral can be expressed in terms of surface integral thus $\oint_V (E\times H) n\, ds$

where n is unit vector

and comparing eqn (1)

$$-\frac{\partial u}{\partial t} = \oint (E\times H) n\, ds \longrightarrow (5)$$

$$\oint p\, ds = \oint (E\times H) n\, ds \Rightarrow \boxed{p = E\times H} \longrightarrow (6)$$

Scanned with OKEN Scanner

UNIT-IV Electromagnetic waves - (1)
Maxwell's equations. Major (9)

→ Basic Laws of Electricity and Magnetism :-

The basic equations of electricity and magnetism ~~which we~~ are in the following 4 equations.

1) $\oint E \cdot ds = \frac{q}{\epsilon_0}$ ⟶ (1)

This is Gauss's Law electricity. which states that the electric flux through a closed surface is equal to the net charge enclosed by the surface divided by the permitivity constant ϵ_0.

2) $\oint B \cdot ds = 0$ ⟶ (2)

This law is Gauss's Law for magnetism. This states that the magnetic flux through a closed surface is zero.

3) $\oint E \cdot dl = \frac{-d\phi_B}{dt}$ ⟶ (3)

This is Faraday's Law of Electromagnetic Induction. This Law states that an electric field is produced by changing magnetic field.

4) $\oint E \cdot dl = \mu_0 i$ ⟶ (4)

This is Ampere's Law for magnetic field due to Steady current. this Law States that the amount of workdone in carrying a unit magnetic pole one

one around a closed arbitrary path linked with the current is μ_0 times the current i.

→ **Plane Electromagnetic wave equation :-**

Let us apply Maxwell's eq's to a homogeneous isotropic dielectric medium.

As dielectric medium has infinite resistance to the current, its conductivity is zero i.e $j = 0$ and $\rho = 0$. Hence

$$j = 0, \quad \rho = 0, \quad D = K\epsilon_0 E, \quad E = \epsilon E \text{ and } B = \mu_0 \mu_r H = \mu H.$$

Hence Maxwell's equations for a dielectric medium becomes

$$\nabla \cdot E = 0 \quad \text{——— (1)}$$

$$\nabla \cdot B = 0 \quad \text{——— (2)}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad \text{———→ (3)}$$

$$\nabla \times B = \mu \epsilon \frac{\partial E}{\partial t} \quad \text{——— (4)}$$

eleminating E from eq (3) and (4), for this taking curl of eq (4) we get

$$\nabla \times \nabla \times B = \nabla \times \mu \epsilon \frac{\partial E}{\partial t} = \mu \epsilon \left(\nabla \times \frac{\partial E}{\partial t} \right)$$

$$= \mu \epsilon \frac{\partial}{\partial t} (\nabla \times E)$$

$$= \mu \epsilon \frac{\partial}{\partial t} \left(-\frac{\partial B}{\partial t} \right) \quad \left(\because \nabla \times E = -\frac{\partial B}{\partial t} \text{ from eq (3)} \right)$$

$$= -\mu \epsilon \frac{\partial^2 B}{\partial t^2}.$$

Thus $\nabla \times \nabla \times B = -\mu \epsilon \frac{\partial^2 B}{\partial t^2}$ ⟶ (5)

We know that

$$\nabla \times \nabla \times B = \nabla(\nabla \cdot B) - \nabla^2 B$$

$$= \nabla(0) - \nabla^2 B.$$

$$\nabla \times \nabla \times B = -\nabla^2 B \quad \text{from eq (2)}$$

⟶ (6)

Substituting value of $\nabla \times \nabla \times B$ from eq (6) in eq (5) we get

$$-\nabla^2 B = -\mu \epsilon \frac{\partial^2 B}{\partial t^2}$$

$$\nabla^2 B = \mu \epsilon \frac{\partial^2 B}{\partial t^2} \longrightarrow (7)$$

Illy from eq (3) we can show that

$$\nabla^2 E = \mu \epsilon \frac{\partial^2 E}{\partial t^2} \longrightarrow (8)$$

eq (7) & (8) represents the relation b/w the Space and time variation of magnetic field B and electric field E. These are called wave equations for B and E. The general wave equation is represented by

$$\nabla^2 y = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}. \longrightarrow (9)$$

Where v is velocity of wave and y is ~~am~~ its amplitude

$$v = \frac{1}{\sqrt{\mu \epsilon}}$$

(4) Unit - IV Major 9

→ **Electromagnetic waves in Conducting Media:-**

we know that Maxwells equations are

$\operatorname{div} D = \rho, \quad \operatorname{div} B = 0.$

$\operatorname{curl} H = J + \frac{\partial H}{\partial t}$ and $\operatorname{curl} E = \frac{-\partial B}{\partial t}$. → (1)

Consider a homogeneous isotropic conducting medium of permittivity ϵ, permeability μ and conductivity α. Then

$J = \alpha E, \quad D = \epsilon E$ and $B = \mu H$ → (2)

In case of conducting media $\rho = 0$.

So Maxwells equations reduce to

$\operatorname{div} E = 0, \quad \operatorname{div} H = 0, \quad \operatorname{curl} H = \alpha E + \epsilon \frac{\partial E}{\partial t}.$

and $\operatorname{curl} E = -\mu \frac{\partial H}{\partial t}$ → (3)

Taking the curl of curl E equation we get

$$\nabla \times (\nabla \times E) = \nabla \times \left(-\mu \frac{\partial H}{\partial t}\right) = -\mu \frac{\partial}{\partial t}(\nabla \times H)$$

$$= -\mu \frac{\partial}{\partial t}\left[\alpha E + \epsilon \frac{\partial E}{\partial t}\right]$$

$$= -\alpha\mu \frac{\partial E}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2}. \quad \left(\because \nabla \times H = \alpha E + \epsilon \frac{\partial E}{\partial t}\right)$$

Unit - IV Major 9

We know that $\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 E$

$$= \nabla(0) - \nabla^2 E = -\nabla^2 E.$$

$$\therefore -\nabla^2 E = -\sigma\mu \frac{\partial E}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E = \sigma\mu \frac{\partial E}{\partial t} + \mu\epsilon \frac{\partial^2 E}{\partial t^2} \longrightarrow (4)$$

Illy $\nabla^2 H = \sigma\mu \frac{\partial H}{\partial t} + \mu\epsilon \frac{\partial^2 H}{\partial t^2} \longrightarrow (5)$

The solution of eq (4) is of the form

$$E = E_0 \exp\{-i(\omega t - k^* \cdot r)\} \longrightarrow (6)$$

Where k^* is the complex wave vector.

Substituting eq (6) in eq (4) we have

$$(-k^{2*} + i\sigma\mu\omega + \mu\epsilon\omega^2)E = 0$$

Since this result is valid for any arbitrary E

Hence $k^{2*} - i\sigma\mu\omega - \mu\epsilon\omega^2 = 0$

$$k^{2*} = \epsilon\mu\omega^2 \left[1 + \frac{i\sigma}{\epsilon\omega}\right] \longrightarrow (7)$$

As the propagation constant k is complex, it can be expressed as

$$k^* = \alpha + i\beta$$

$$k^{*2} = \alpha^2 - \beta^2 - i2\alpha\beta \longrightarrow (8)$$

Comparing eq (7) and (8) we have

6

Unit - IV Major 9.

$$\alpha^2 - \beta^2 = \mu\epsilon\omega^2 \longrightarrow (9)$$

$$2\alpha\beta = \mu\sigma\omega \longrightarrow (10)$$

from eq (9) and (10)

$$\alpha^2 - \frac{\mu^2\sigma^2\omega^2}{4\alpha^2} = \mu\epsilon\omega^2$$

$$\alpha^4 - \mu\epsilon\omega^2\alpha^2 - \frac{\mu^2\sigma^2\omega^2}{4} = 0.$$

This gives

$$\alpha^2 = \frac{\mu\epsilon\omega^2 \pm \sqrt{[(\mu\epsilon\omega^2)^2 + \mu^2\sigma^2\omega^2]}}{2}$$

$$\alpha = \pm\omega\sqrt{\left[\frac{\mu\epsilon}{2}\right]}\left[1 \pm \sqrt{\left\{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2\right\}}\right]^{1/2} \longrightarrow (11)$$

In the limit $\sigma \longrightarrow 0$, $\mu \longrightarrow \mu_0$, $\epsilon \longrightarrow \epsilon_0$

eq (7) becomes

$$k^{2*} = \mu_0\epsilon_0\omega^2 \text{ and } k^* = \alpha$$

$$\alpha \rightarrow \omega\sqrt{[\mu_0\epsilon_0]}$$

IIIly α in terms of β, we have

$$\beta = \omega\sqrt{\left[\frac{\mu\epsilon}{2}\right]}\left[\sqrt{\left\{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2\right\}} - 1\right]^{1/2} \longrightarrow (12)$$

eq (6) becomes

$$E = E_0 \exp\left[-i\omega t - (\alpha + i\beta)\,n.r\right]$$

$$E = E_0 \left[\exp(-\beta n \cdot r) \exp\{-i(\omega t - \alpha n \cdot r)\} \right] \longrightarrow (13)$$

$$v = \frac{\omega}{\alpha} = \sqrt{\left[\frac{2}{\mu \epsilon}\right] \left[\left\{ 1 + \frac{\sigma^2}{\epsilon \omega} \right\} + 1 \right]^{-1/2}}$$

$$v = \sqrt{\frac{2\epsilon}{\mu \alpha}} \longrightarrow (14)$$

Thus the velocity of wave, being inversely proportional to α is very small is good conducth.

Hertz Experiment :-

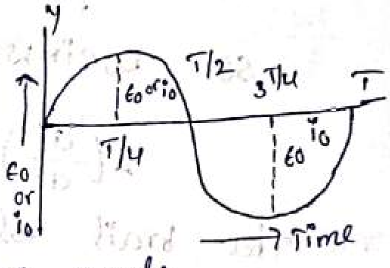
~~UNIT-III~~ Unit-V

Varying Ey Alternating current

Alternating current:- The AC or Alternating current is defined as one which passes through a cycle of changes at regular intervals. The wave form is shown in fig and its mathematically represented by

$$i = i_0 \sin\omega t$$

(1)



or $\epsilon = \epsilon_0 \sin\omega t$

where 'ωt' is called phase.

From the fig we see that one cycle consists of two half cycles during one of its +ve and other one is negative.

The value of current at any incident 'T' is given by $i = i_0 \sin\omega t$

The average value of sinosoidel wave over one complete cycle is given by

$$i_{avg} = \frac{\int_0^T i_0 \sin\omega t}{\int_0^T dT} = \frac{\frac{-i_0}{\omega}\left|\cos\omega t\right|_0^T}{T}$$

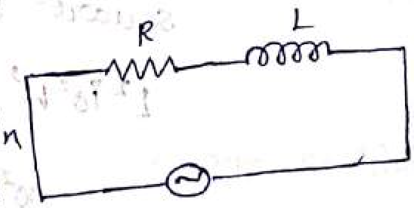
[where $\omega t = 2\pi$]

$$= \frac{-i_0}{\omega T}\left|\cos\frac{2\pi T}{T}\right|_0^T$$

$$= \frac{-i_0}{\omega T}\left[\cos 2\pi - \cos 0\right] = \frac{-i_0}{\omega T}[1-1] = 0$$

Alternating current ckt containing RL, or relation b/w current and voltage in LR:-

consider a circuit containing a resistance 'R' and inductance in series 'L' in series connected to a source of alternating EMF $\epsilon = \epsilon_0 \sin\omega t$ as shown in figure.



Let i be the instaneous value of the circuit due to change in current and induced emf is set up in the inductance is given by $-L\left(\frac{di}{dt}\right)$

now effective emf in the circuit is

$$\left[E_0 \sin\omega t - L\left(\frac{di}{dt}\right)\right]$$

According to ohm's law is equal to Ri

so $E_0 \sin\omega t - L\left(\frac{di}{dt}\right) = Ri \longrightarrow (1)$

$$L\left(\frac{di}{dt}\right) + Ri = E_0 \sin\omega t \longrightarrow (2)$$

The trail solution for eqn (2) is $i = i_0 \sin(\omega t - \phi) \longrightarrow (3)$

where i_0 and ϕ are constants to find out these values diff eqn (3)

$$\frac{di}{dt} = i_0 \omega \cos(\omega t - \phi)$$

substitute these value in Eq (2) we get

$$L i_0 \omega \cos(\omega t - \phi) + R i_0 \sin(\omega t - \phi) = E_0 \sin\omega t \longrightarrow (4)$$

$$L i_0 \omega \cos(\omega t - \phi) + R i_0 \sin(\omega t - \phi) = E_0 \sin(\omega t - \phi + \phi)$$

$$L i_0 \cos(\omega t - \phi) + R i_0 \sin(\omega t - \phi) = E_0 (\sin(\omega t - \phi)\cos\phi + \cos(\omega t - \phi)\sin\phi)$$

Equations, the co-efficient of $\cos(\omega t - \phi)$ $\sin(\omega t - \phi)$ on both sides

$$L i_0 \omega = E_0 \sin\phi \longrightarrow (5)$$

$$R i_0 = E_0 \cos\phi \longrightarrow (6)$$

square & add (5) & (6)

$$L^2 i_0^2 \omega^2 + R^2 i_0^2 = E_0^2$$

$$i_0^2 \left[L^2 \omega^2 + R^2\right] = E_0^2$$

$$i_0 = \frac{E}{\sqrt{L^2 \omega^2 + R^2}} \longrightarrow (7)$$

Divide (5) and (6)

$$\frac{E_0 \sin\phi}{E_0 \cos\phi} = \frac{L i_0 \omega}{R i_0}$$

3

$$\tan\phi = \frac{L\omega}{R}$$

$$\phi = \tan^{-1}\left(\frac{L\omega}{R}\right) \longrightarrow (8)$$

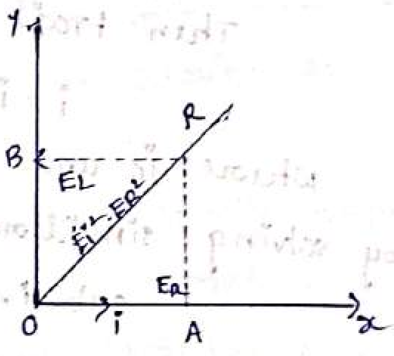
Eq (7) & (8) represents the current in the circuit and the current lags in face behind the emf by an angle ϕ is equal to $\tan^{-1}\frac{\omega L}{R}$ $\phi = \tan^{-1}\frac{\omega L}{R}$

The impedence $Z = \frac{E_0}{I_0} = \sqrt{R^2+\omega^2 L^2}$

Vector diagram:-

The vector diagram can be clotted by consider voltage across resistance always remain in face with the current but voltage across inductance let over to current at 90°.

Let E_R and E_L be the magnitudes of voltage across resistance and inductances $E_R = iR$ and $E_L = i\omega L$ are shown in fig (8)



Hence $E^2 = E_L^2 + E_R^2$

$$(iZ)^2 = (i\omega L)^2 + (Ri)^2$$

$$E = iZ$$

$$i^2 Z^2 = i^2\omega^2 L^2 + R^2 i^2$$

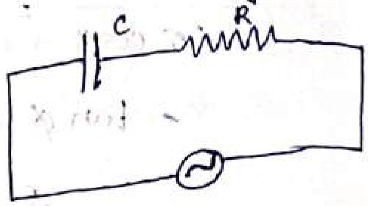
$$Z^2 = R^2 + \omega^2 L^2$$

$$\therefore Z = \sqrt{R^2+\omega^2 L^2}$$

$$\tan\phi = \left(\frac{E_L}{E_R}\right) = \left(\frac{\omega L}{R}\right)$$

$$\phi = \tan^{-1}\left[\frac{\omega L}{R}\right]$$

Alternating current containing capacitance and Resistance (CR) in series :- (Relation b/w current and voltage in CR CKT):- consider a circuit containing a resistance 'R' and capacitance 'C' in series connected to the source of emf $\epsilon = \epsilon_0 \sin\omega t$ as shown in fig. Let q be the charge on the capacitor at any incident of time 't' and i be the current in the circuit the potential difference across the capacitor is q/c



4

Then the effective emf in the circuit induced emf $\left[\epsilon_0 \sin\omega t - \frac{q}{c}\right]$

From Ohm's Law this is equal to Ri

hence $\epsilon_0 \sin\omega t - \frac{q}{c} = Ri$

$$Ri + \frac{q}{c} = \epsilon_0 \sin\omega t \longrightarrow (1)$$

diff eq (1) $\left[\frac{dq}{dt} = i\right]$

$$R\frac{di}{dt} + \frac{1}{C}\frac{dq}{dt} = \epsilon_0 \omega \cos\omega t \longrightarrow (2)$$

$$R\frac{di}{dt} + \frac{i}{C} = \epsilon_0 \omega \cos\omega t \longrightarrow (3)$$

Then trail solution for eqn (3) is

$$i = i_0 \sin(\omega t - \phi) \longrightarrow (4)$$

where i_0 and ϕ are constants to be determined by solving similarly as above we get

sub in eqn (3)

$$i_0 = \frac{\epsilon_0}{\sqrt{R^2 + \left(\frac{1}{\omega^2 C^2}\right)}} \longrightarrow (5)$$

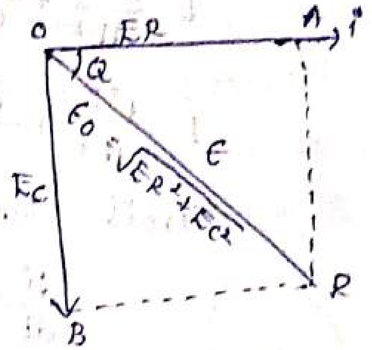
$$Z = \frac{\epsilon_0}{i_0} = \sqrt{R^2 + \left(\frac{1}{\omega^2 C^2}\right)} \longrightarrow (6)$$

$$\phi = \tan^{-1}\left(\frac{1}{\omega C R}\right) \longrightarrow (7)$$

<u>Vector diagram</u>:—

The value of impedence and phase values can be obtained by vector diagram. Let E_R

and E_c be the magnitudes of voltages across resistance 'R' and condensor 'C' then



$$E^2 = E_R^2 + E_c^2 \longrightarrow (1)$$

$$E = iz$$

$$(iz)^2 = (Ri)^2 + \left(\frac{i}{wc}\right)^2$$

$$z = R^2 + \left(\frac{1}{wc}\right)^2$$

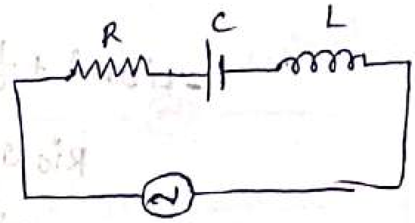
$$= R^2 + \frac{1}{w^2c^2}$$

$$z = \sqrt{R^2 + \frac{1}{w^2c^2}}$$

$$\tan\phi = \frac{E_c}{E_R} = \frac{1}{wCR}$$

$$\phi = \tan^{-1}\left(\frac{1}{wCR}\right)$$

AC ckt containing inductance (L) capacitance (c) and resistor (LCR series):-



Let Alternating emf $E = E_0 \sin wt$ we applied to the circuit containing inductance 'L' capacitance 'c' and resistance 'R' in series as shown in fig.

Let 'i' be the current in the circuit at any time 'T' and q be the charge on the capacitor then the potential difference across the capacitor is q/c and the emf induced due to the self inductance L is $-L\frac{di}{dt}$

Hence $\left[E_0 \sin wt - \frac{q}{c} - L\left(\frac{di}{dt}\right)\right]$

According to ohm's Law

$$E_0 \sin wt - \frac{q}{c} - L\frac{di}{dt} = Ri$$

$$L\frac{di}{dt} + \frac{q}{c} + Ri = E_0 \sin wt \longrightarrow (1)$$

diff Eq ①

$$L\frac{d^2i}{dt^2} + \frac{1}{C}\cdot\frac{dq}{dt} + R\frac{di}{dt} = E_0\omega \cos\omega t \qquad \left[\frac{dq}{dt} = i\right]$$

$$L\frac{d^2i}{dt^2} + \frac{i}{C} + R\frac{di}{dt} = E_0\omega\cos\omega t \longrightarrow ②$$

Trail solution $i = i_0 \sin(\omega t - \phi)$

$$\left.\begin{aligned}\frac{di}{dt} &= i_0\omega\cos(\omega t - \phi)\\ \frac{d^2i}{dt^2} &= -i_0\omega^2\sin(\omega t - \phi)\end{aligned}\right\} \longrightarrow ③$$

6

sub ③ in eqn ②

$$\Rightarrow -Li_0\omega^2\sin(\omega t - \phi) + \frac{1}{C}i_0\sin(\omega t - \phi) + Ri_0\omega\cos(\omega t$$
$$= E_0\omega\cos(\omega t - \phi + \phi$$

$$\Rightarrow -Li_0\omega^2\sin(\omega t - \phi) + \frac{1}{C}i_0\sin(\omega t - \phi) + Ri_0\omega\cos(\omega t - \phi$$
$$= E_0\omega\left[\cos(\omega t - \phi)\cos\phi - \sin(\omega t - \phi)\sin\phi\right.$$

$$-Li_0\omega^2 + \frac{1}{C}i_0 = -E_0\sin\phi \longrightarrow ④$$

$$Ri_0\omega = E_0\omega\cos\phi \longrightarrow ⑤$$

squaring and adding ④ & ⑤

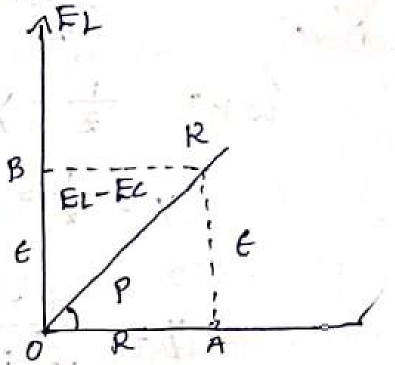
$$L^2i_0^2\omega^2 + \frac{i_0^2}{C^2} + R^2i_0^2\omega^2 = E_0^2\omega^2$$

$$i_0^2\left[L^2\omega^2 + \frac{1}{C^2} + R^2\omega^2\right] = E_0^2\omega^2$$

$$i_0 = \frac{E_0\omega}{\sqrt{L^2\omega^2 + \frac{1}{C^2} + R^2\omega^2}} \Rightarrow i_0 = \frac{E_0}{\sqrt{LR^2 + \left(\omega L - \frac{T}{\omega L}\right)^2}} \longrightarrow ⑥$$

Vector diagram :-

The vector diagram with series and LCM series is shown in fig. The potential difference across the resistance is represented by a vector OA - Ri_0. The voltage across the inductance $E_L = \omega L i_0$, which is 90° behind the current the resultant E can be obtained by vector diagram



7

$$E_0^2 = (OR)^2$$

$$= (Ri_0)^2 + i_0^2 (\omega L - \frac{1}{\omega C})^2$$

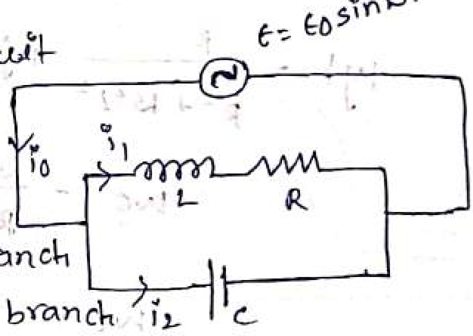
$$= i_0^2 [R^2 + (\omega L - \frac{1}{\omega C})^2]$$

$$Z = \frac{E_0}{i_0} = [R^2 + (\omega L - \frac{1}{\omega C})^2]^{1/2}$$

fig $\tan \phi = \dfrac{\{\omega L - \frac{1}{\omega C}\}}{R}$

Parallel resonant circuit :-

A parallel resonent circuit is shown in fig. An inductance 'L' and resistance 'R' are connected in series in one branch and capacitor 'C' in another branch a source alternating emf is connected in the ckt.



$E = E_0 \sin \omega t$ The current from the Generator is i_0 and is distributed in two branchs as shown in fig.

From kirchoff's Law

$i_0 = i_1 + i_2$ Let Z be the impendence of the circuit

The impedence of the resistance and inductance branch is $Z_1 = (R + j\omega L)$

The impedence of the condenser branch

$$Z_2 = \frac{1}{j\omega C}$$

The above branchs are in parallel. Hence the resultant impedence is

$$\Rightarrow \frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

6

$$\Rightarrow \frac{1}{Z} = \frac{1}{(R+j\omega L)} + \frac{1}{1/j\omega C}$$

$$\frac{1}{Z} = \frac{1}{(R+j\omega L)} + j\omega C \longrightarrow (1)$$

WKT $\frac{1}{Z} = Y$.

$$Y = \frac{1}{Z} = \frac{1}{R+j\omega L} + j\omega C \longrightarrow (2)$$

$$Y = \frac{R-j\omega L}{(R-j\omega L)(R+j\omega L)} + j\omega C \quad (j^2=-1)$$

$$= \frac{R-j\omega L}{R^2+\omega^2L^2} + j\omega C$$

$$|Y| = \frac{R}{R^2+\omega^2L^2} + j\left[\omega C - \frac{\omega L}{R^2+\omega^2L^2}\right] \longrightarrow (3)$$

$$\omega C - \frac{\omega L}{R^2+\omega^2L^2}$$

$$\omega C = \frac{\omega L}{R^2+\omega^2L^2}$$

$$C = \frac{L}{R^2+\omega^2L^2}$$

$$C(R^2+\omega^2L^2) = L$$

$$CR^2+C\omega^2L^2 = L$$

$$C\omega^2L^2 = L-CR^2$$

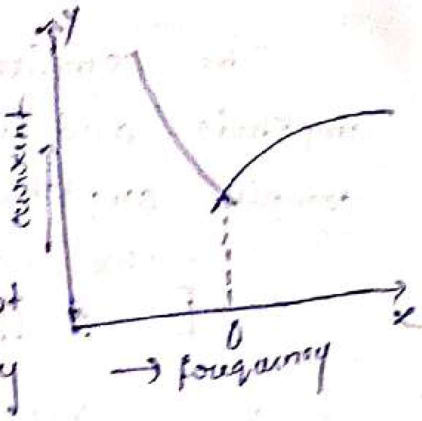
$$\omega^2 = \frac{L-CR^2}{CL^2}$$

$$\omega = \left\{\frac{L-CR^2}{CL^2}\right\}^{1/2}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\left[\frac{L-CR^2}{CL^2}\right]^{1/2} \longrightarrow (4)$$

vector diagram .

At this frequency the admittance is minimum and hence current is minimum such a frequency is called resonant frequency. the circuit frequency is known as parallel resonant.



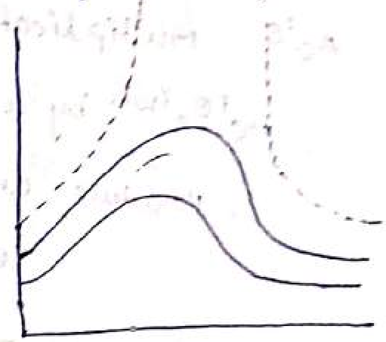
The variation of current in the frequency is shown in figure.

9

quality factor or Q-factor of a ckt :-

quality factor is an element of efficiency of energy stored in an inductor as capacitor when alternating current is applied this is defined as 2π times of energy stored to the per average

$$Q = 2\pi \frac{\text{Energy stored}}{\text{Energy Loss per period}}$$



$$Q = 2\pi f \frac{\text{Energy stored}}{\text{Power loss in one sec}}$$

$$= 2\pi f \frac{\frac{1}{2} L i_0^2}{\frac{1}{2} i_0^2 R}$$

$$= \frac{2\pi f L}{R} = \frac{\omega L}{R}$$

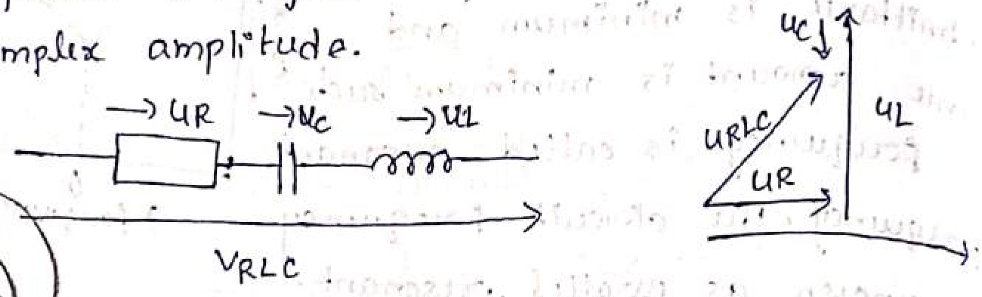
amplitude of ascillating current

$Q = \frac{\omega L}{R}$ = amplifide of oscillator current

by this Q factor the Q factor is a measure of current amplification.

Phasor:- phasor is a complex number represents a sinusoided function. whose amplitude A, angular frequency ω and initial phase θ are time invariant.

The complex constant which depends on amplitude and phase is known as phasor (or) complex amplitude.

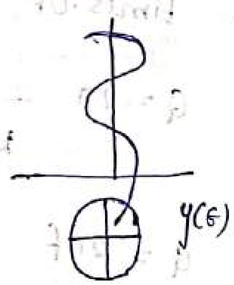


10

fig ①

As shown in fig example of series RLC, ckt or refractive phasor diagram for a specific ω

The function $Ae^{i(\omega t+\theta)}$ is called the analytic representation of $A\cos(\omega t+\theta)$ in fig ②. depics it as a rotating vector, in complex plane. But the term phasor usally implies the static complex number $Ae^{i\theta}$ multiplication of a phasor



$Ae^{i\theta}e^{i\omega t}$ by a constant $Be^{i\phi}$

produce another phasor

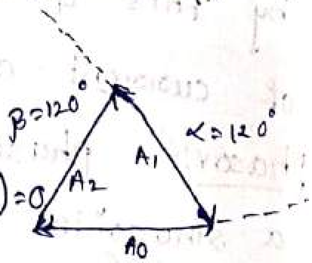
$$\text{ie } [Ae^{i\theta}\cdot Be^{i\phi}]\, e^{i\omega t}$$

$$= (ABe^{i(\theta+\phi)})\, e^{i\omega t}$$

$$= AB\cos[\omega t + (\theta+\phi)]$$

In electronics $Be^{i\phi}$ represents an impedence, which is independent of the time multiplying a phasor current by an impedence produces a phasor voltage from the diagram, the angle between each phasor to the next is $120°$ ($2\pi/3$ radians) or one third of a wave length $\lambda/3$.

In other words



$$\cos\omega t + \cos(\omega t + 2\pi/3) + \cos\left(\omega t - \frac{2\pi}{3}\right) = 0$$

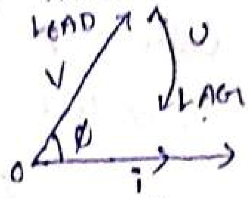
Ex:- ohm's law for resistor, $V = IR$

Scanned with OKEN Scanner

Ohm's Law for resistors, Inductors and capacitors

$V = IZ$ where Z is complex impendence

The rotating frame picture using phasor can be powerful tool to understand analog modulations and frequency modulations



11

$$x(t) = Re\left(Ae^{j\theta} \cdot e^{j2\pi f_0(t)}\right)$$

In bracket term is rotating vector in complex plane. The phasor has length A rotates anticlock wise at a rate of f_0 revolutions per second at time $t=0$ make an angle of θ w.r.t +ve real axis. The wave from $x(t)$ can be viewed as a projection of this vector on to the real axis.

Power in ac circuit :-

consider the case of an ac current in which applied voltage $E = E_0 \sin wt$ results in a current $i = i_0 \sin wt\ (wt+\theta)$. The phase angle θ will be +ve or -ve depending on the circuit is inductive or capacitive

The instantaneous power is given by

$$P = Ei = E_0 i_0 \sin wt \cdot \sin(wt+\theta) \longrightarrow ①$$

Since $\sin A \cdot \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

and $\cos(-A) = \cos A$ we get

$$P = \frac{1}{2} E_0 i_0 \left[\cos\theta - \cos(2wt+\theta)\right] \longrightarrow ②$$

From the above eqn the instantaneous power P consists of a sinusoidal from $-\frac{1}{2} E_0 i_0 \cos(2wt+\theta)$ which has an average value of zero and a constant term $\frac{1}{2} E_0 i_0 \cos\theta$.

Then the average value of P is given by

$$P = \frac{1}{2} E_0 i_0 \cos\theta = Ei\cos\theta \longrightarrow (A)$$

Where $E = E_0/\sqrt{2}$ and $i = i_0/\sqrt{2}$ are the effective values of alternating voltage and current $\cos\theta$ is called power factor.

Equation (A) can be generalised in the following manner

True power = Apparent power × power factor.

12

4. Electromagnetic Induction

In 1831 Faraday discovered whatever the magnetic lines of force the induced current flows in the ckt and induced emf gives rise to such current is called induced electromotive force and this phenomenon is called electromagnetic induction.

→ Faraday's Laws of Electromagnetic Induction :-

The following are the two laws of electromagnetic induction
① whenever the mag flux linked with a ckt is changed an e.m.f is induced in the ckt.
② The magnitude of induced e.m.f is directly proportional to the -ve rate of variation of magnetic flux ~~lik~~ linked with the ckt, If ϕ_B be the mag flux linked with ckt at any instant and e be the induced emf then

$$e = -\left(\frac{d\phi_B}{dt}\right) \longrightarrow (1)$$

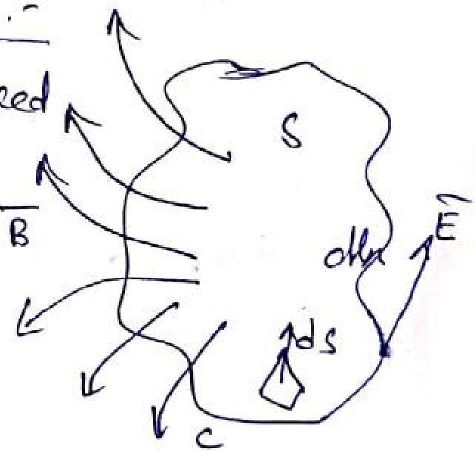
This Law is also known as Neumann's Law.
If there are N turns in the coil, then

$$e = -N\left(\frac{d\phi_B}{dt}\right)$$

15

Vector form of Faraday's Law :-

Consider that magnetic field is produced by a Stationary magnet & current carrying coil. Suppose there is a closed ckt C of any shape which ~~the~~ encloses a surface S. in the field as shown in fig.



Let B be the magnetic flux density in the neighbourhood of the ckt. The magnetic flux through a small area ds will be B.ds

Now the flux through the entire ckt is

$$\phi_B = \int_S B \cdot ds \longrightarrow (2)$$

When mag flux is changed an electric field is induced around the ckt. The line integral of the electr field gives the induced emf in the closed ckt. Thus

16

$$e = \oint E \cdot dl \longrightarrow (3)$$

Where E is electric field at an element dl of the ckt. Substituting the values of e and ϕ_B from eq (3) and (2) in eq (1) we have

$$\boxed{\oint E \cdot dl = -\frac{d}{dt}\int_S B \cdot ds.} \longrightarrow (4)$$

This is the integral form of faradays law.

According to eq (4) the line integral of electric field around any closed ckt is equal to the -ve rate of change of magnetic flux through the ckt.

Further by Stokes theorem, we have

$$\oint E \cdot dl = \int_S (\nabla \times E) ds \longrightarrow (5)$$

from eq (4) and (5)

$$\int_S (\nabla \times E)\, ds = -\frac{d}{dt}\int_S B \cdot ds.$$

$$\int_S (\nabla \cdot \times E) = -\int_S \frac{\partial B}{\partial t}.$$

It follows that $\nabla \times E = -\frac{\partial B}{\partial t}$.

$$\boxed{\text{curl } E = -\frac{\partial B}{\partial t}}$$ (17)

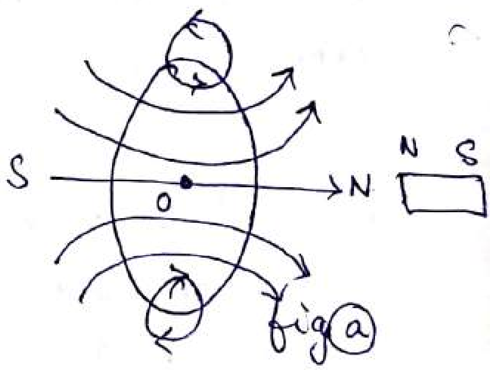
This is differential form of Faraday's Law.

→ Lenz's Law:-

This Law is based on the principle of conservation energy. When the applied flux density B. in a closed ckt is increasing the emf & current induced in the closed ckt is in such a direction to produce a field which tends to decrease B.

The induced current is in a direction such that it produces a magnetic flux tending to oppose the original change of flux tending to keep the total flux const in ckt.

Suppose the north pole of mag is moved towards a coil connected to a galvanometer, as shown in fig. As the magnet is pushed towards the ckt and induced current is setup in coil. The induced current produces its own magnetic field. Now the coil behaves as a magnet.



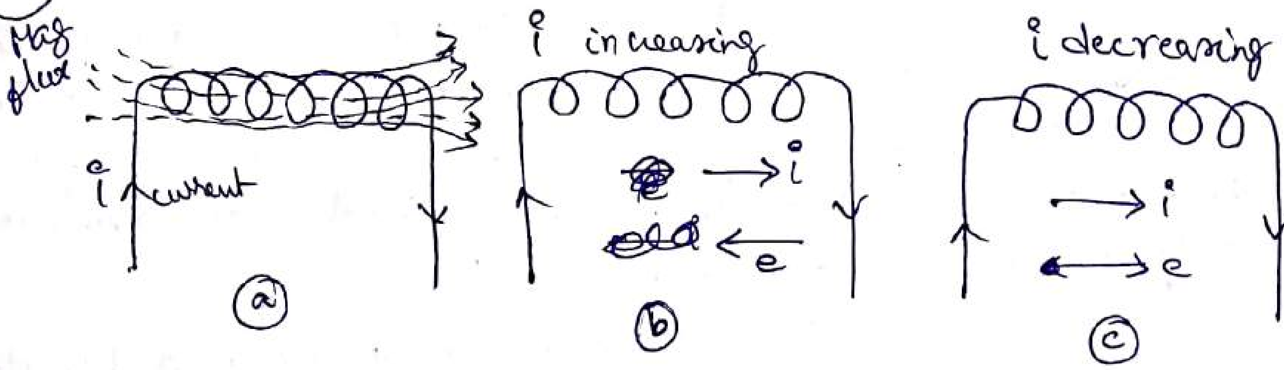
fig(a)

The face of the coil towards the north pole of the mag becomes a north pole. So there will be a force of repulsion b/w them. Due to this force of repulsion the motion of the magnets is opposed. This causes a change of magnetic flux in the coil. Thus the direction of induced current is ~~sh~~ such that as to oppose the motion of the magnet.

→ Self and Mutual inductance :-

The phenomenon of Self induction was discovered by J. Henry in 1832. When a current flows in a coil magnetic field is set up in it as shown in fig (a)

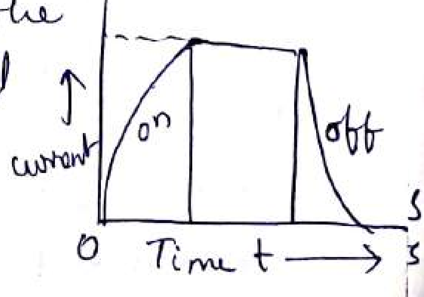
18



If the current passing through the coil changes with time an induced emf is setup in the coil. By len's law the direction of induced emf is such as to oppose the change in current.

When the current is increasing the induced emf is against the current as shown in fig (b) and when current is decreasing it is in the direction of current as shown in fig (c). So the induced emf opposes any change of the original current. This phenomenon is called self induction.

When the current in a coil is switched on self induction opposes the growth of the current and when current is switched off the self induction opposes the decay of current as shown in fig.



coefficient of Self induction:-

The total magnetic flux ϕ_B linked with the Coil is proportional to the current i flowing in it. i.e

$$\phi_B \propto i$$

19

$$\boxed{\phi_B = Li} \longrightarrow (1)$$

Where L is a const. called the coefficient of Self induction & Self inductance of the Coil. when $i = 1$, $\phi_B = L$.
Hence the coefficient of self induction is numerically equal to the mag flux liked with the coil, when unit current flows through it.
The emf induced in the coil is given by

$$e = -\frac{d\phi_B}{dt} \qquad \{ \because \phi_B = Li \}.$$

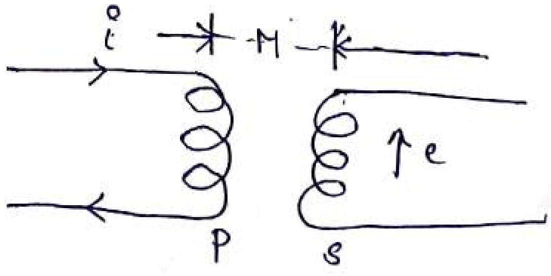
$$= -\frac{d(Li)}{dt} = -L\frac{di}{dt}. \longrightarrow (2)$$

The -ve sign indicates that the induced emf is in Such a direction as to oppose the change.

when $\frac{di}{dt} = 1$, $\boxed{e = -L.}$

Coefficient of Mutual Induction:-

Consider two coils placed near to each other as shown in fig. when the current passed in the primary coil P there is a change of magnetic flux



linked with it and an induced emf is set in the Secondary coil. The total induction through the Secondary is the difference b/w the inductions due to primary and secondary.

Let a current i amp in primary p produces a mag ~~fl~~ flux ϕ_B in the secondary S. The flux linked with the secondary is proportional to the current in primary. Thus

20

$$\phi_B \propto i$$

$$\boxed{\phi_B = Mi} \longrightarrow (3)$$

Henry

where M is const. Called coefficient of Mutual induction or Mutual inductance of two coils. The e.m.f induced in the secondary S is given by

$$e = \frac{-d\phi_B}{dt}$$

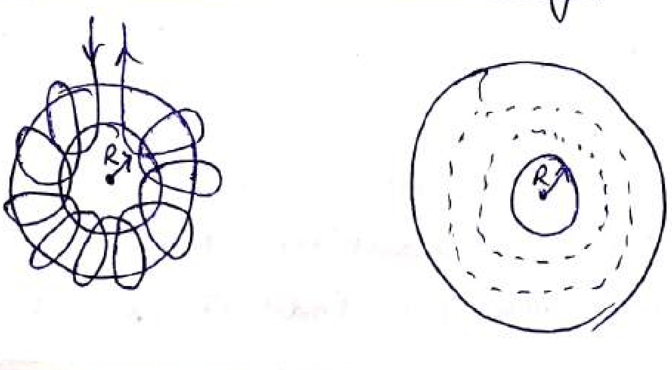
$$\boxed{e = -\frac{d}{dt} Mi = -M\frac{di}{dt}} \longrightarrow (4)$$

The eq (3) & (4) enables us to define the mutual inductance of the following two ways.

(1) It is the flux linked with a circuit due to a unit current flowing through the other.

(2) It is the emf induced in the ckt, when the rate of decay of current in the other ckt is unity.

→ Coefficient of Coupling:-



consider two coils very close to each other and having no of turns n_1 and n_2 as shown in fig. Let i_1 and i_2 be the currents flowing in two coils. By definition of self inductance

$$L_1 = \frac{n_1 \phi_1}{i_1} \longrightarrow (1) \qquad , \qquad L_2 = \frac{n_2 \phi_2}{i_2} \longrightarrow (2)$$

21

Here ϕ_1 and ϕ_2 are the magnetic fluxes linked with coils 1 and 2 due to their own currents i_1 and i_2.
Where a current i passed through first coil there will be a magnetic flux linked with each turn of second coil, it is ϕ_{21}, so the total flux linked with n_2 turns of second coil will be $n_2 \phi_{21}$.

Suppose the coefficient of mutual inductance of ~~first~~ second coil due to current in the ~~second~~ first coil be M_{21}, then

$$n_2 \phi_{21} = M_{21} i_1 \longrightarrow (3)$$

Illy the coefficient of Mutual inductance of first coil due to current in the second coil be M_{12}, then

$$n_1 \phi_{12} = M_{12} i_2 \longrightarrow (4)$$

If the two coils are wound on the same core, so that the centre flux set up by either coil links with all the turns of the other, then the coupling is said to be perfect. Now

$$\phi_{12} = \phi_2 \quad \text{and} \quad \phi_{21} = \phi_1 \longrightarrow (5)$$

The situation is known as maximum flux linkage situation. In this situation applying the reciprocity theorem we have

$$M_{12} = M_{21} = M_{maximum} \longrightarrow (6)$$

Applying conditions 5 and 6 to eq's 3 and 4 we get

$$n_2\phi_1 = M_{max} i_1 \longrightarrow (7)$$

$$n_1\phi_2 = M_{max} i_2 \longrightarrow (8)$$

Multiply eq 7 and 8 we get

$$M_{max}^2 i_1 i_2 = n_1 n_2 \phi_1 \phi_2$$

$$M_{max}^2 = \left[\frac{n_1\phi_1}{i_1}\right] \times \left[\frac{n_2\phi_2}{i_2}\right] \longrightarrow (9)$$

Substitute the values of eq 1 and 2 in eq 9 we get

$$M_{max}^2 = L_1 L_2.$$

$$\boxed{M_{max} = \sqrt{L_1 L_2}} \longrightarrow (10)$$

eq 10 is true only when the whole of the effective flux from one coil links with other.

& ∴ $\boxed{M = K\sqrt{L_1 L_2}} \longrightarrow (11)$

where K is called coefficient of coupling b/w two coils. Its value varies from 0 to 1 depends upon the shape of the two coils.

if $K = 1$, coupling is tight — i.e no leakage flux.
if $K = 0$ there is no coupling b/w the coils
if $K > 0$ and $K < 1$ there is optimum coupling.

→ Calculation of Self inductance of a long Solenoid :-

Consider a long air core Solenoid of length l meter and uniform cross-section area A meter. Let n be the no of turns per meter. Suppose a current i amp flows through it. The magnetic field inside the Solenoid is given by

$$B = \mu_0 n i \quad \text{weber/meter}^2.$$

23

μ_0 – permeability cost.

$\therefore$ Magnetic flux through each turn

$$\phi_B = BA = \mu_0 n i A \text{ weber.}$$

Now the magnetic flux linked with all the turns of Solenoid $= \mu_0 n i A \times N$ weber turn.

where N is equal to total no of turns in the Solenoid.

$$= \mu_0 n i A \times n l \qquad \{\because N = nl\}$$

$$= \mu_0 n^2 i A l.$$

The Self inductance of the Solenoid is therefore

Li = Total flux linked with the Solenoid.

$$Li = \mu_0 n^2 i A l$$

$$\boxed{L = \mu_0 n^2 A l} \text{ henry} \longrightarrow (1)$$

where n is no. of turns per unit length.

In terms of total no of turns N of the Solenoid

$$L = \mu_0 \left(\frac{N}{l}\right)^2 A l.$$

$$\boxed{L = (\mu_0 N^2 A)/l} \text{ henry} \longrightarrow (2)$$

→ Energy Stored in a magnetic field :-

consider a very long Solenoid of length l and Cross - sectional area A. When a current flows in it, a mag field is established. This field is uniform inside and neglegible outside, So the volume associated with the magnetic field is Al.

The amount of workdone in establishing a current i_0 in the Solenoid is $(\frac{1}{2}) L i_0^2$,

Where L is the inductance of Solenoid. The workdone is stored as energy in the magnetic field.

$$U = \text{energy Stored} = \frac{1}{2} L i_0^2 .$$

The inductance of the Solenoid is given by

$$L = \mu_0 n^2 A l .$$

Where n is no. of turns in Solenoid ~~is given by~~ per meter

$$\therefore\ U = \frac{1}{2} (\mu_0 n^2 A l) i_0^2 = \frac{1}{2} \frac{(\mu_0 n i_0)^2}{\mu_0} A l .$$

The magnetic field inside the Solenoid

$$B = \mu_0 n i_0 .$$

$$\therefore\ U = \frac{1}{2} \frac{B^2}{\mu_0} A l .$$

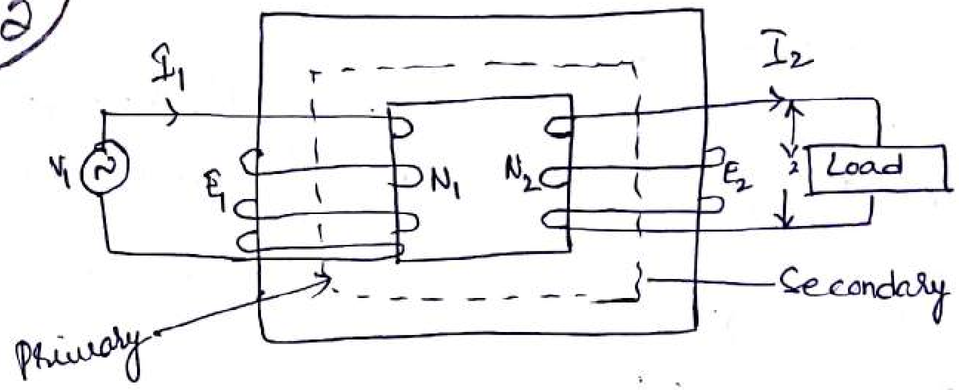
This energy is uniformly distributed throughout the volume Al of the magnetic field. So the energy density (energy per unit volume) u in magnetic field is given by

$$u = \frac{U}{Al} = \frac{1}{2} \cdot \frac{B^2}{\mu_0} \cdot \frac{Al}{Al} = \frac{1}{2} \frac{B^2}{\mu_0} \ \text{joule/m}^3$$

$$u = \frac{B^2}{2\mu_0} \ \frac{\text{Joule}}{\text{meter}^3} .$$ It should be noted the indutance in a ckt plays the same role as the mass & inertia does in mechanical motion.

Transformer :- ~~Energy~~ :- Energy - losses - efficiency :- A transformer is an A.C Static device which transfers electric power from one ckt to another. It can rise & lower the voltage in ckt but with a decrease & increase in current. fig Shows the basic transformer.

25



Any transformer has two coils electrically insulated but magnetically linked. By magnetic link means the two coils are ~~wond~~ wound on a Common core. The energy from one coil is transformed to other coil by means of magnetic coupling. The coil which receives energy from an A.C Source called Primary (P) and the coil which delivers the energy to load is called Secondary (S)

A transformer operates on the principle of mutual induction, When an alternating voltage is applied to the primary an A.C is set up in it. from Faradays Law of electromagnetic induction

$$e = M\left(\frac{dI}{dt}\right)$$

emf induced = - rate of change of flux linkage.

$$e = -N\frac{d\phi}{dt} \text{ volt} \longrightarrow (1)$$

e - instantaneous emf induced in volts.

ϕ - " value of mutual flux

N - No of turns in the coil.

Let N_1 and N_2 be the no. of turns of p and S. The eq ① can be written as

$$e_1 = -N_1 \frac{d\phi}{dt}.$$

from eq $\phi = \phi_m \sin 2\pi f t$,

then $e = -N_1 \frac{d}{dt}(\phi_m \sin 2\pi f t)$

28

$$= -N_1 \phi_m 2\pi f \cos 2\pi f t.$$

$$e_1 = N_1 \phi_m 2\pi f \sin(2\pi f t - \pi/2) \text{ volts.}$$

Thus the emf induced in a coil lags the flux induced by an angle of 90° & $\pi/2$ radians,

$$E_{max} = N_1 \phi_m 2\pi f \text{ Volt.}$$

$$E_{rms} = \frac{E_{max}}{\sqrt{2}} = E_1$$

$$E_1 = 4.44\, \phi_m N_1 f \text{ volts} \longrightarrow ③$$

Illy $E_2 = 4.44\, \phi_m N_2 f$ volts $\longrightarrow ④$

from eq ③ & ④

$$\frac{E_1}{E_2} = \frac{4.44\, \phi_m N_1 f}{4.44\, \phi_m N_2 f} = \frac{N_1}{N_2} = a \longrightarrow ⑤$$

where a is called tranformation ratio, and it gives information about transformer type.

If $N_2 > N_1$ – then $a < 1$ – Step up transformer

$N_1 > N_2$ – then $a > 1$ – Step down transformer.

For ideal transformer

Input V × A = output V × A.

$$V_1 I_1 = V_2 I_2$$

$$\boxed{\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{1}{a}.} \longrightarrow (6)$$

27

Efficiency of transformer

$$\eta = \frac{\text{output power}}{\text{Input power}} = \frac{V_2 I_2}{V_1 I_1}.$$

Principle - Lenz's Law.

Energy losses:-

The following are the major sources of energy loss in a transformer.

(1) Copper loss:- is the energy loss in the form of heat in the copper coils of a transformer. This is due to joule heating of conducting wires.

(2) Iron loss :- is the energy loss in the form of heat in the iron core of the transformer. This is due to formation of eddy currents in iron core. It is minimized by taking laminated cores.

(3) Leakage of magnetic flux:- occurs inspite of best insulations. Therefore, rate of change of magnetic flux linked with each turn of $S_1 S_2$ is less than the rate of change of magnetic flux linked with each turn of $P_1 P_2$.

(4) Hysteresis loss:- is the loss of energy due to repeated magnetization and demagnetization of

Applications :- power is generated at around 11 KV in a powerhouse. for transmission we utilize higher rating say around 110 KV & 200 KV. Again to supply voltage at load centres it has to be reduced to say 6.6 KV and then supply it to ~~custo~~ customer to 220 V. for this we need Step-down transformers. This is major application of a transformer.

28

iron core when A.C is fed to it.

(5) Magneto Striation :- humming noise of a transformer.

Step-up	Step-down	Step up	Step down
(1) This is when no of o/p coils is greater than the no of i/p coils which means that there will be a greater o/p voltage as opposed ~~bef~~ to i/p voltage	(1) In this when the no of o/p coils is less than the no of i/p coils which means that there will be less o/p voltage as opposed to i/p voltage	(4) Transformer will step up the voltage to b/w 110 KV to 1000 KV for transmission over long distance at very low cost	(4) The transformer will stepped down the voltage to the 12 KV to 34.5 KV range for local distribution in the home.
(1) A transformer that has more turns on the secondary than the primary side of transformer will increase the i/p voltage is called Step-up	(1) A transformer that has more turns on the primary than the secondary side of the transformer will decrease the i/p voltage is called step-down	(5) used for the production of X-rays and NEON advertisement	(5) used for welding purpose
(2) Converts low voltage A.C into high voltage A.C. It increases the voltage	(2) Converts high voltage A.C into low voltage A.C. It reduces the voltage	(6)	(6) obtaining large current.
(3) $E_s/E_p = \frac{N_s}{N_p} > 1$	(3) $E_s/E_p = N_s/N_p < 1$	all a.c operations	

→ Diff b/w step-up, Step down transformer.

VI Sem | paper VI | UNIT-I

1. Atomic & Molecular ~~Spect~~ physics.

~~Atom~~ consits electrons, proton and neutron. It was not known at that time how the +ve & -ve charges are distributed in an atom. To explain the structure of atom lot of experiments done and are came as atomic models. The various models are as follows:

1. Thomson's plum pudding model
2. Rutherford's Nuclear model
3. Bohr's model
4. Sommerfield's relativistic model
5. Vector model and
6. Wave mechanical model.

I UT
28th – III – III
31 – III – IV

Drawbacks of Bohr's Theory:-

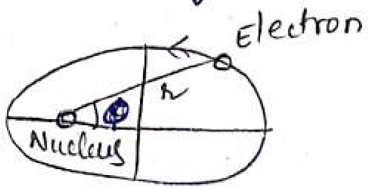
The Bohr's theory fails to explain the following facts

1. The theory could not account the spectra of atoms more complex than hydrogen
2. It does not explain the distribution and arrangement of electrons in atom
3. It does not explain the experimentally observed variation in intensity of spectral lines of an element.
4. It cannot be used to calculate & how to use selection rules which apply to them.
5. This theory fails to accounting fine structure of sepectral lines.
6. It does not explain quantative explanation of ~~bo~~ chemical bonding
7. It fails to explain the Application result of ~~applie~~ electric and magnetic field applied to the atom.

Sommerfield's elliptical orbits :- (i)

To overcome Bohr's atomic model shortcommings 1921 Sommerfield introduces an idea of motion of electron in elliptical orbits and taking into consideration the ~~vibration~~ variation of mass with velocity. And this model is known as Sommerfeld's relativistic model.

According to Sommerfeld the electron moving around the nucleus under an inverse square force (planet around sun). In elliptical orbit the position of electron at any time may be fixed by two co-ordinates r and ϕ as shown in fig.



The tangential velocity of the electron can be resolved into two components — one along radius vector called radial velocity and the other perpendicular to radius vector called the transverse velocity.

Corresponding to these velocities the electron has two momenta one along radius vector called as radial momentum and along ⊥er to radius vector known as azimuthal momentum.

Sommerfeld introduced two quantum numbers n_r and n_ϕ and $n_r + n_\phi = n$

where n_r – radial quantum number

n_ϕ – angular or azimuthal quantum number

n – is known as principal quantum number.

length of semi major axis — principal quantum number n

" semiminor " — azimuthal quantum n_ϕ

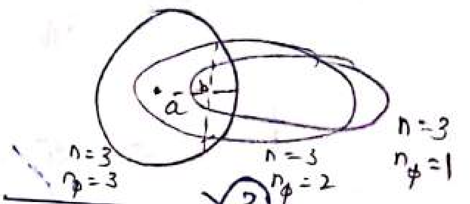
- Sommerfeld assumed $\oint p_r\, dr = n_r h \longrightarrow (1)$ (2)

$$\oint p_\phi\, d\phi = n_\phi h \longrightarrow (2)$$

Where p_r and p_ϕ are components of linear momentum p of the electron along radial direction and transverse direction n_r, n_ϕ are integers, $\oint$ indicates that the integration is to be carried out over one cycle of the motion,

we know that the angular momentum p_ϕ remains const during the motion. From eq (2) we have

$$\oint_0^{2\pi} p_\phi\, d\phi = p_\phi \oint_0^{2\pi} d\phi = p_\phi \cdot 2\pi = n_\phi h$$



& $p_\phi = n_\phi \dfrac{h}{2\pi} \longrightarrow (3)$

In an elliptical orbit the radius vector is a variable quantity. Hence

$$\oint p_r\, dr = n_r h.$$

Now consider the value of p_r $(p = mv)$

$$p_r = m\left(\frac{dr}{dt}\right) = m\left(\frac{dr}{d\phi}\right) \times \left(\frac{d\phi}{dt}\right)$$

$$= m\left(\frac{dr}{d\phi}\right)\left(\frac{p_\phi}{mr^2}\right) \qquad \left[\because p_\phi = mr^2\left(\frac{d\phi}{dt}\right)\right]$$

$$= \left(\frac{p_\phi}{r^2}\right)\left(\frac{dr}{d\phi}\right)$$

Substitute the value of p_r, we get

$$\oint \frac{p_\phi}{r^2} \cdot \left(\frac{dr}{d\phi}\right) dr = n_r h$$

$$\oint \frac{p_\phi}{r^2} \left(\frac{dr}{d\phi}\right)\left(\frac{dr}{d\phi}\right) d\phi = n_r h$$

$$p_\phi \oint \left[\frac{1}{r}\frac{dr}{d\phi}\right]^2 d\phi = n_r h \longrightarrow (4)$$

(3) Using polar co-ordinates (r, ϕ) in an ellipse, we have

$$\frac{1}{r} = \frac{1+\epsilon\cos\phi}{a(1-\epsilon)^2} \longrightarrow (5)$$

where a is semi major axis and ϵ is eccentricity.
diff eq (5) we get

$$\frac{1}{r^2}\frac{dr}{d\phi} = \frac{\epsilon\sin\phi}{a(1-\epsilon)^2} \longrightarrow (6)$$

divide eq (6) by eq (5) we get

$$\frac{1}{r}\frac{dr}{d\phi} = \left(\frac{\epsilon\sin\phi}{1+\epsilon\cos\phi}\right) \longrightarrow (7)$$

From eq (4) and (7) we get

$$p_\phi \oint \left[\frac{\epsilon\sin\phi}{1+\epsilon\cos\phi}\right]^2 d\phi = n_r h$$

$$p_\phi \int_0^{2\pi} \frac{\epsilon^2\sin^2\phi}{(1+\epsilon\cos\phi)^2} d\phi = n_r h \longrightarrow (8)$$

Let $u = \epsilon\sin\phi$ and $v = \frac{1}{(1+\epsilon\cos\phi)}$

$du = \epsilon\cos\phi\, d\phi$ and $dv = \frac{\epsilon\sin\phi\, d\phi}{(1+\epsilon\cos\phi)^2}$

Now using $\int u\,dv = uv - \int v\,du$ we get

$$p_\phi\left[\left\{\frac{\epsilon\sin\phi}{(1+\epsilon\cos\phi)}\right\}_0^{2\pi} - \left\{\int_0^{2\pi}\frac{\epsilon\cos\phi\, d\phi}{(1+\epsilon\cos\phi)}\right]\right\} = n_r h$$

$$\frac{n_\phi h}{(1-\epsilon^2)^{1/2}} - n_\phi h = n_r h, \Rightarrow \frac{n_\phi}{(1+\epsilon^2)^{1/2}} - n_\phi = n_r.$$

4

$$n_r + n_\phi = \frac{n_\phi}{(1-\epsilon^2)^{1/2}}$$

$$(1-\epsilon^2)^{1/2} = \frac{n_\phi}{n_r + n_\phi} = \frac{n_\phi}{n}$$

$$(1-\epsilon^2)^2 = \frac{n_\phi^2}{n^2} \longrightarrow (9)$$

we know that $(1-\epsilon^2) = \left(\frac{b^2}{a^2}\right) \longrightarrow (10)$

Where a, b are the semi-major and semi minor axes of the ellipse respt.

Compare eq (9) (10)

$$\frac{n_\phi^2}{n^2} = \frac{b^2}{a^2} \quad \& \quad \boxed{\frac{n_\phi}{n} = \frac{b}{a}} \longrightarrow (11)$$

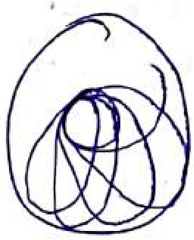
eq (11) give condition of allowed elliptical orbits. only allowed elliptical orbits are ~~other~~ those for which the ratio of minor to major axes is the ratio of n_ϕ / n.

→ **Sommerfeld Relativistic Correction**

Sommerfeld Relativistic correction :-

Sommerfeld pointed out that the velocity of electron moving in an elliptical orbit varies at different parts of the orbit. It is maximum when the electron is nearest to the nucleus and min when it is farthest from the nucleus.

This implies that effective mass of electron will be different at different parts of its orbit. Sommerfeld has made such calculations and showed that the path of the electron is not a closed ellipse but a complicated curve known as rosette - aprocessing ellipse whose major axist precesses slowly in the plane of the ellipse about an axis through one of the foci as shown in fig.



5

The expression of energy of an electron in a hydrogen like atom for a particular State characterised by quantum number n and n_ϕ after relativistic correction can be shown to be

$$E_n = -\frac{mz^2e^4}{8\epsilon_0^2 \epsilon^2 h^2 \times} \left[1 + \frac{\alpha^2 z^2}{n} \left(\frac{1}{n_\phi} - \frac{3}{4n} \right) \right]$$

where

$$\alpha = \frac{e^2}{2\epsilon_0 ch} \simeq \frac{1}{317}$$

α is called as sommerfeld fine structure constant.

The energy expression shows that the energy depends not only on the principal quantum no but also on azimutal quantum no. $n = 1, n_\phi = 1$

for $n = 2, n_\phi = 1$, $n_\phi = 2$ (2 energy levels + ------)

This accounts why a single line of hydrogen spectrum should appear as a group of closely associated lines. Thus the introduction of relativity correction accounts for the fine structure of the spectral lines.

→ Vector atom model:-

1923

This model is an extension of Bohr-Sommerfeld atom model.

Concept:-

The two concepts which characterise the vector atom model and differentiate it from other model are

a) The concept of quantisation of direction i.e Spatial quantisation

b) The concept of spining electron.

(6)

Space quantisation:-

Bohr's theory:- electron moves in circular orbit, the radius of the orbit remains fixed, the electron has only one degree of freedom.

Sommerfeld:- elliptical orbit, the electron possesses two degree of freedom. (n_r, n_ϕ are sufficient to describe the electron orbit.

In vector atom model:- which is based on quantum theory the orbits are assumed to be quantised in magnitude and direction. and such a spatial quantisation makes the orbit vector quantised.

According to the rule of space quantisation the electron orbit can only set itself in certain discrete positions with respect to the field direction and not in all positions. The angle b/w field direction and the direction which is ⊥er to the plane of orbit ~~can only set~~ is taken as θ.

The electron orbit can only set itself in certain discrete positions with respect to the field direction and not in all positions making all possible angles with the field direction as suggested by classical theory. This is known as spatial quantisation

b) Spinning electron:-

In alkali Spectra it was observed that many lines consists of a group of lines close to each other.

To explain the multiple character of spectral lines, Goudsmit 1925 forward hypothesis of electron spin.

According to this hypothesis the electron revolves not only in an orbit round the nucleus, but also about its own axis. The electron has two types of motions orbital motion and spin motion. And here a new quantum number ie spin Quantum number is introduced.

The orbital and spin motions are quantised not only in magnitude but also in direction according to the concept of spatial quantization.

(7)

Quantum Numbers Associated with vector atom Model:-

The quantum no associated with each of the electrons in a given atom are the following.

(1) Principal or total quantum no (n) :- electron belongs.

This quantum no belongs to the principal orbit to which electron belongs to. In terms of principal quantum no n, the energy of the electron and its distance from the nucleus for hydrogen atom are given by

$$E_n = \frac{-13.6}{n^2} eV. \quad \text{and} \quad r_n = 0.529 n^2 A^\circ$$

n can have only non-zero +ve integral values $n = 1, 2, 3 - - - \infty$ and the energy levels or shells are $\underset{1}{K}, \underset{2}{L}, \underset{3}{M}, \underset{4}{N}, \underset{5}{O}, \underset{6}{P}, \underset{7}{Q}$. The no of electron shells is limited to $2n^2$.

(2) Orbital quantum number (l) :- shape of orbital

This quantum number defines the shape of the orbital occupied by the electron and the orbital angular momentum of the electron. It may have integral ~~number~~ values from 0 to (n-1). Each value refers to an energy sub-shell. $l = 0, 1, 2, 3$ are designated as s, p, d, f etc. When $n = 1, l = 0$; $n = 2, l = $ 3 values $- l = 0, l = 1$

The orbital angular momentum P_l for the electron is equal to $lh/2$ where l denotes the angular momentum in unit of $h/2\pi$. where $l = \sqrt{l(l+1)}$

$$P_l = \sqrt{[l(l+1)]}\,\frac{h}{2\pi}, \quad h \text{ plancks Const.}$$

Here l and n_ϕ (azimutal quantum no) are related as $l = (n_\phi - 1)$

8

3 Spin quantum number: (S)

This quantum no has been introduced to account for the spin of the electrons about their own axis. The magnitude is always $\frac{1}{2}$ (half). The electron can spin clock wise & anticlock wise $+\frac{1}{2}, -\frac{1}{2}$.

Two electron have same spin - have parallel spin
" " opposite " - paired up spins.

The angular momentum is denoted by P_s and is given by $s(h/2\pi)$

$$P_S = \sqrt{s(s+1)}\cdot\frac{h}{2\pi}. \quad \left\}\ \because s = \sqrt{s(s+1)}\right.$$

4 Total angular quantum number (j):-

This quantum number represents the resultant angular momentum of the electron due to both orbital and spin motions. It is vector sum of l and s. $j = (l + \frac{1}{2})$. The total angular momentum of the electron is $P_j = jh/2\pi$ & $P_j = \sqrt{(j+1)}\,\frac{h}{2\pi}$.

5 Magnetic orbital quantum number m_l:-

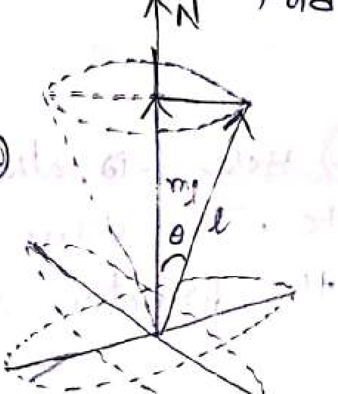
When an atom is placed in a strong magnetic field the electrons with same values of principal quantum number (n) and orbital quantum number (l), To account for one more quantum number (m_l) known as magnetic orbital quantum number has been introduced. This is produced by Zeeman effect and Stark effect.

Zeeman effect refers to the splitting of spectral lines due to magnetic field while Stark effect refers to the splitting of spectral lines due to electric field.

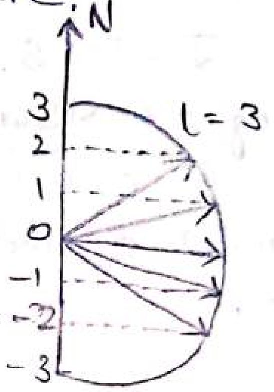
According to the rules of space quantization the orientations of l with H and this projection of l in the field direction is known as magnetic orbital quantum number.

Consider the case when the vector l is inclined at an angle θ with the direction as shown in fig (a). The projection $m_l = l\cos\theta$. In the absence of magnetic field five orbitals are equivalent energy and they are said to be five fold degenerate. For $l = 3$, $m_l = +3, +2, +1, 0, -1, -2, -3$ i.e there are seven permitted orientations as shown in fig (b) and is said to be seven fold degenerate.

fig (a)



9



n
l
s
j – angular quantum no.
ml
ms
mj

(6) Magnetic spin quantum number m_s :- $(2s+1)$

The numerical value of m_s is the projection of spin vector S on the field direction. By analogy with the orbital vector m_s can have any of the $(2s+1)$ values from $-s$ to $+s$ excluding zero. However s is always equal to $\frac{1}{2}$ and never zero. This means that m_s can have only two values $+\frac{1}{2}$ and $-\frac{1}{2}$.

(7) Magnetic total angular momentum quantum number m_j :-

It is projection of the total angular momentum vector j on the field direction and m_j can have half integral values. i.e $(2j+1)$ values from $+j$ to $-j$. This shows that magnetic field contributes energy to the electron and each electron splits into $(2j+1)$ values from $+j$ to $-j$ exclude zero. explain splitting

→ Coupling Schemes :-

Generally two types of coupling known as Russell - Saunders & L-S Coupling and j-j coupling occurs which described below.

(1) L-S Coupling :-

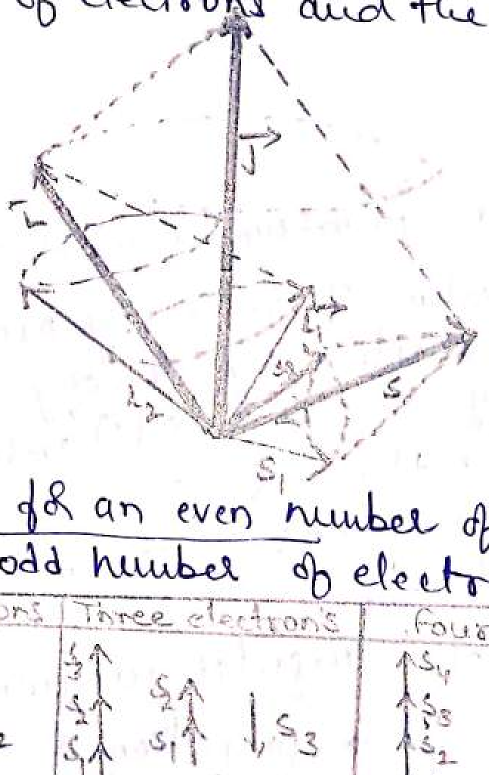
This type of coupling occurs most frequently and hence is known as normal coupling. In this coupling all the orbital angular momentum vectors l of the electrons combine to form resultant vector L and all the spin angular momentum vectors S. Now L and S vectors combine to form a vector J which represents the total angular momentum of the atom. We can represent coupling as

$$L = (l_1 + l_2 + l_3 + \cdots)$$
$$S = (s_1 + s_2 + s_3 + \cdots)$$

10

and $$J = L + S.$$

L-S Coupling is shown in fig (a) Here L is always an integer zero i-e 0, 1, 2, 3 ... etc. The value of S depends upon the number of electrons and the direction of their spin vector.



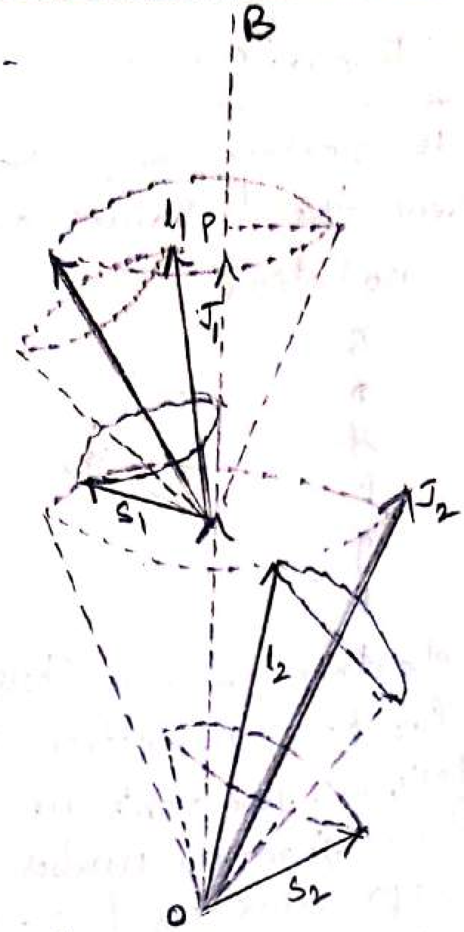
S is an integer for an even number of electrons and half integer for an odd number of electrons.

Two electrons		Three electrons		Four electrons		
$s_1\uparrow s_2\uparrow$	$s_1\downarrow$ $s_2\uparrow$	$s_1\uparrow s_2\uparrow s_3\uparrow$	$s_1\uparrow s_2\uparrow$ $s_3\downarrow$	$s_1\uparrow s_2\uparrow s_3\uparrow s_4\uparrow$	$s_1\uparrow s_2\uparrow s_3\uparrow$ $s_4\downarrow$	$s_1\uparrow s_2\uparrow$ $s_3\downarrow s_4\downarrow$
$S = 1$,	0	3/2	1/2	2	1	0

Now J = integer (0, 1, 2, 3 --- etc) when S is integer
J = half integer ($1/2$, $3/2$, $5/2$ ---etc) when S is half integer
J is always +ve because it represent total angular momentum of the atom

② J-J Coupling :-

(11)



Sometimes the interaction b/w the spin and orbital vectors in each electron is stronger than the interaction b/w either the spin vectors & orbital vectors of different electrons. In such cases J-J coupling is most suitable than L-S coupling. In J-J coupling each electron is considered separately and its total angular momentum j is obtained by the relation $j = l + s$. Then the total angular momentum J of the atom would be vector sum of all the individual j vectors of the electrons.

Thus $j_1 = (l_1 + s_1)$, $j_2 = (l_2 + s_2)$, $j_3 = (l_3 + s_3)$ - - -

and $J = (j_1 + j_2 + j_3) + - - - -$

$= \Sigma j$

Pure j-j coupling is seldom found. In most of the known cases L-S coupling is effective.

Scanned with CamScanner

→ Zeeman Effect and its experimental arrangement:-

In 1896, prof. Zeeman discovered that when a Source of radiation, giving line Spectrum is placed in a magnetic field, the spectral lines are split up into a no of component lines, symmetrically distributed about the original line, Doublets, triplets and even more complex Systems are observed, This is known as ~~Zeeman~~ Zeeman effect.

If the mag field is very strong each Spectral line is split up into two components in longitudinal view and in 3 components in transverse view This is known as Normal Zeeman effect,

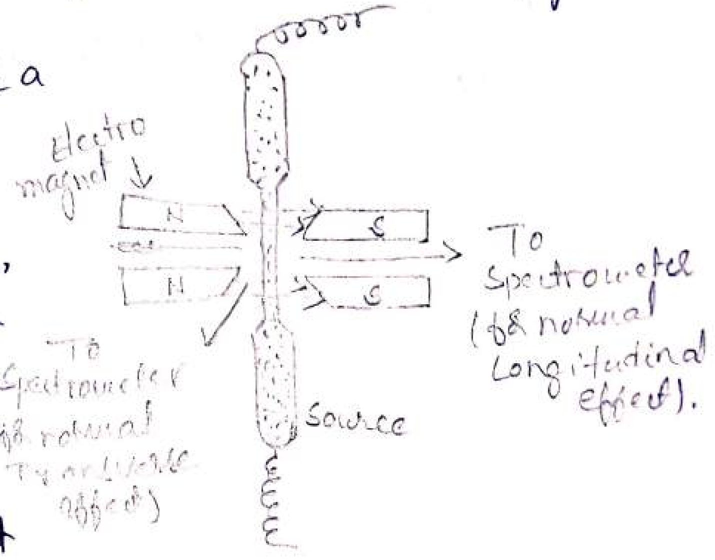
When the magnetic field is weak, each line splits into more than 3 components, This is known as Anomalous zeeman effect.

Experimental Arrangement:-

12

The experimental arrangement for observing normal zeeman effect is shown in fig ①.

The Source of radiation Such as Sodium flame or a mercury are is placed between the pole pieces of powerful electromagnet,



The electromagnet has conical pole pieces and holes are drilled along the length so that light from the source can pass throught it.

Thus the Spectrum can be observed along the direction of magnetic field. The Spectral lines are observed with a high Resolving power Spectroscope. The Spectral lines may also be observed in a direction

$\perp$er to the mag field. three (normal) components are observed. The original line having original freq ν and two components being either side of the central line and equally seperated from it having frequencies $\nu + \Delta\nu$ and $\nu - \Delta\nu$. The central line is polarised and the other are linearly polarised at right angles to the mag field as shown in fig (2).

In the absence of magnetic field being unpolarised, when light is viewed along the direction of mag field the same additional lines with frequencies $\nu + \Delta\nu$ and $\nu - \Delta\nu$ are observed while the central line having freq ν is missing. These two lines are circularly polarised one being clock wise and the other anticlock wise i-e these two lines are polarised in opposite directions as shown in fig (2) by arrows.

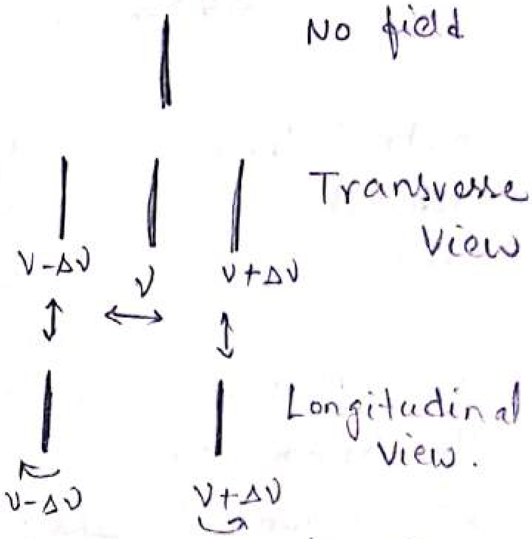


fig (2)

Scanned with CamScanner

...ecentricity. ... the electron orbit and

Stern – Gerlach Experiment :- (14)

(Varification of Space Quantization Concept and electron Spin) :-

In Stern and Gerlach expt a beam of Silver atom is passed through an inhomogeneous magnetic field. The Silver atom beam is produced by heating Silver in a Small electric oven O as shown in fig (a) and passing the beam through slits S_1 and S_2.

The inhomogeneous magnetic field is produced having one of the pole pieces of the magnets flat with a cylindrical groove and other in the form of a knife edge parallel to groove as shown in fig

Scanned with CamScanner

250 → 35

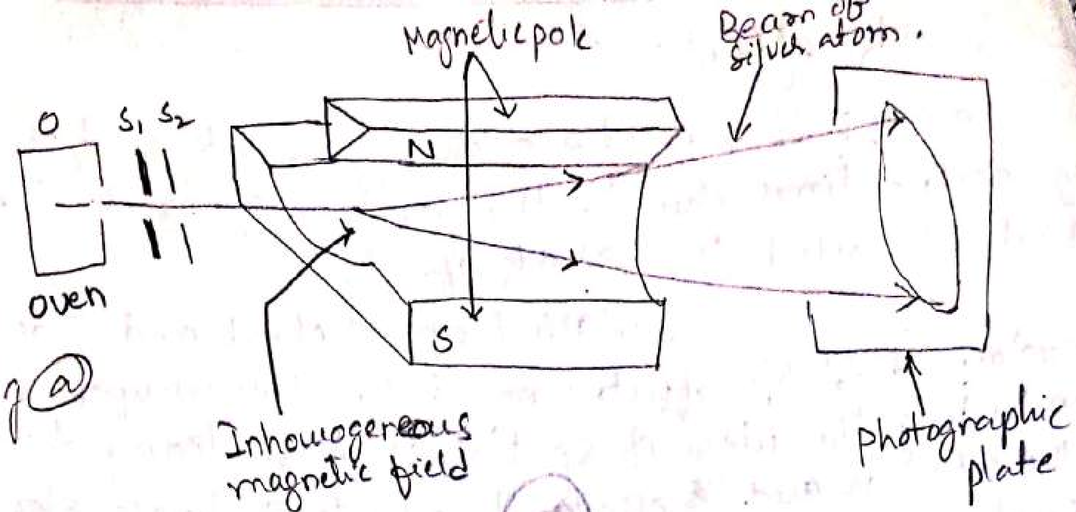


fig (a)

15

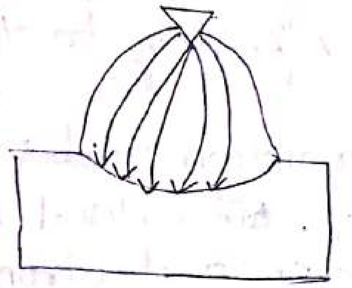
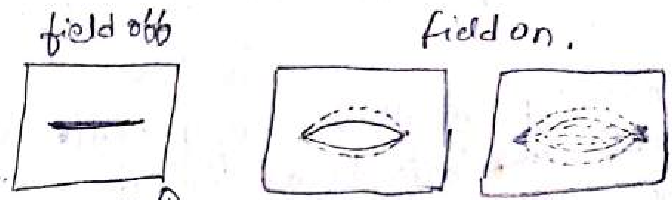


fig (b) Side view of magnet with lines of force.



No field (1) pattern

(2) Actual pattern

(3) classical patter

fig (c)

The intensity of magnetic field increases as we go from the centre towards the upper knife edge pole and decrease as we go below towards the lower pole fig (b). A photographic plate p records the configuration of the beam after its passage through the field.

In the absence of magnetic field a trace of the form of a narrow strip is obtained as shown in fig (c)(1). In the presence of the inhomogeneous magnetic field the strip splits up into two components as shown in fig (c)(2). According to classical concepts the classical shape of the strip ~~in~~ in inhomogeneous field is shown in fig (c)(3).

The splitting of silver beam into two components in inhomogeneous field verifies the existence of electron ~~pattern~~ spin and the postulates of space quantization as shown below.

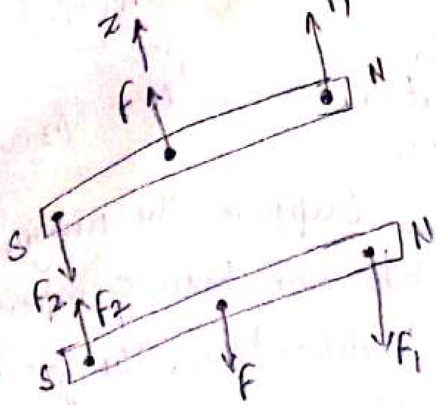


fig (d)

fig (e)

16

According to pauli's ~~expt~~ excultion principal, all the inner shells and subshells are completely filled except outmost
7:47 electron in 5s state. (~~This 5s state electron is responsible for magnetic moment of atom. When all silver atoms possessing a magnetic moment μ_j passes through the inhomogeneous magnetic field they experience different forces depends upon their oreintation~~). The magnetoo experience a resultent force $F_1 - F_2$ as shown in fig(d). This force causes a deflection of silver atom ~~to varying degrees in vertical direction~~ ~~Experimentally only two narrow strips~~ are obtained on the photographic plate and the orientations are shown in fig (e). (~~we Know that μ_j is proportional to angular~~ momentum $\cancel{J}$ and according to ~~space~~ quantisation rules, the magnitude J is given by

$$J = \sqrt{[j(j+1)]}\ h/2\pi$$

There are $(2j+1)$ possible orientations of J
The Strern Gerlach expt shows that $(2j+1) = 2$
& $j = 1/2 \Rightarrow$ Thus $J = \frac{\sqrt{3}}{2} \frac{h}{2\pi}$
Thus we conclude that the electron has a spin angular momentum $S = \sqrt{s(s+1)} \cdot h/2\pi$ where $s = 1/2$

Thus Stern and Gerlach found that the initial beam(s) are split into two distinct parts.

An expression for the amount of deviation produced may be obtained as follows:-

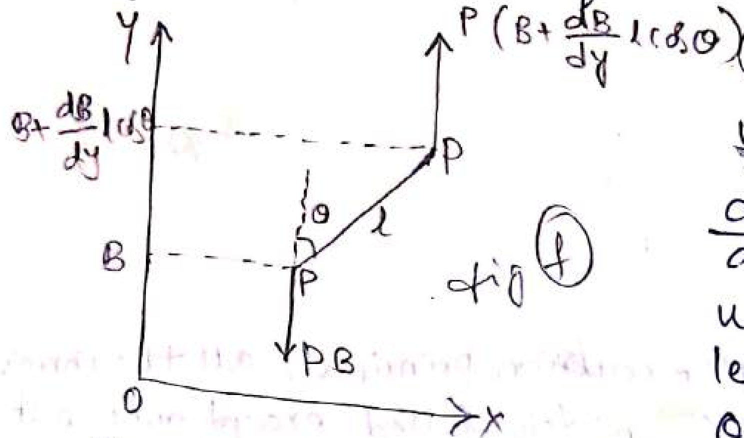


fig (f)

~~Suppose the magnetic field be non-homogeneous along Y-direction~~). The gradient is $\frac{dB}{dy}$ and is +ve. The magnetic moment M, pole strength P and length l inclined at an angle θ as shown in fig (f)

Force on one pole, atomic magnet is PB while other is $P(B+\frac{dB}{dy}\cdot l\cos\theta)$. The force F_y is given by

17

$$F_y = P \cdot l\cos\theta \frac{dB}{dy} = M\cos\theta \frac{dB}{dy} \longrightarrow (1)$$

$Pl = M =$ magnetic moment.

Let silver atom enter into non-homogeneous field with velocity v, and the length L and time t, The displacement ~~dy~~ d_y at time t is given by

$$d_y = \frac{1}{2}\alpha_y t^2 \longrightarrow (2)$$

α_y is acceleration imparted to the atom. Thus $\alpha_y = F_y/m$ where m is mass of atomic ~~m~~ magnet.

$$d_y = \frac{1}{2}\frac{F_y}{m}\cdot t^2 = \frac{1}{2}\frac{F_m}{m}\left(\frac{L}{V}\right)^2 \qquad \left(\because t = \frac{L}{V}\right)$$

Substitute F_y from eq (1)

$$(d)_y = \frac{1}{2}\frac{M\cos\theta}{m}\frac{dB}{dy}\left(\frac{L}{V}\right)^2 \longrightarrow (3)$$

eq (3) was verified by experiment. The theoritical and experimental results are in excellent aggrement. This strongly support the postulates of space quantization and the existence of electron spin.

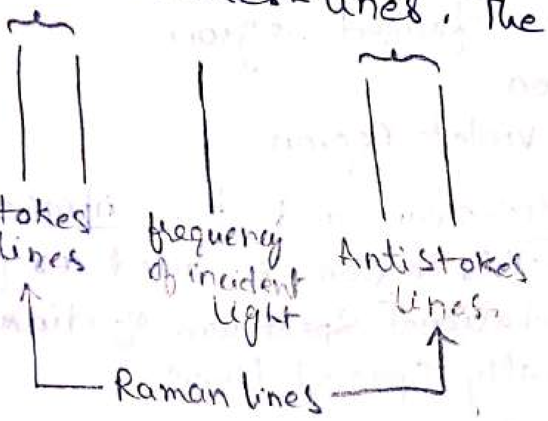
Raman Effect:- & hypothesis, & characteristics

When a monochromatic beam of light is passed through a gas liquid & transparent solid body, a small fraction of the light is scattered in all directions.

In 1928 Sir. C.V. Raman observed that the Spectrum of scattered light consists of frequencies greater and smaller that of the incident beam frequency. This is known as Raman effect. The spectrum of the scattered light is called Raman Spectrum. The new lines are known as Raman lines.

The lines of greater frequency are called as anti-stokes lines while the lines of smaller frequency are called as stokes-lines. The lines are shown in fig.

18



The displacement of the lines are independent of the frequency of incident light but are functions of the scattering substance. Hence Raman displacements are characteristic of scattering Substance. Characteristics of Raman

Differenciate b/w fluoresence and Raman effect:-

In fluoresence the frequency of emitted light is less than that of the incident light because first of all the light is absorbed and then it is re-emitted. incident and absorption ~~fre~~ frequencies are same.

In Raman (effect) scattering the frequency of incident light may be far different from any of the absorption frequencies of the substance.

Scanned with CamScanner

~~Raman Effect:- Experimental Study:-~~

Characteristics of Raman effect:-

(1) The freq's of Raman lines depend on the freq of incident light

(2) The displacement of Raman lines depends on nature of scattering substance.

(3) The lines of greater freq are called as anti stokes and smaller freq is called as stokes lines.

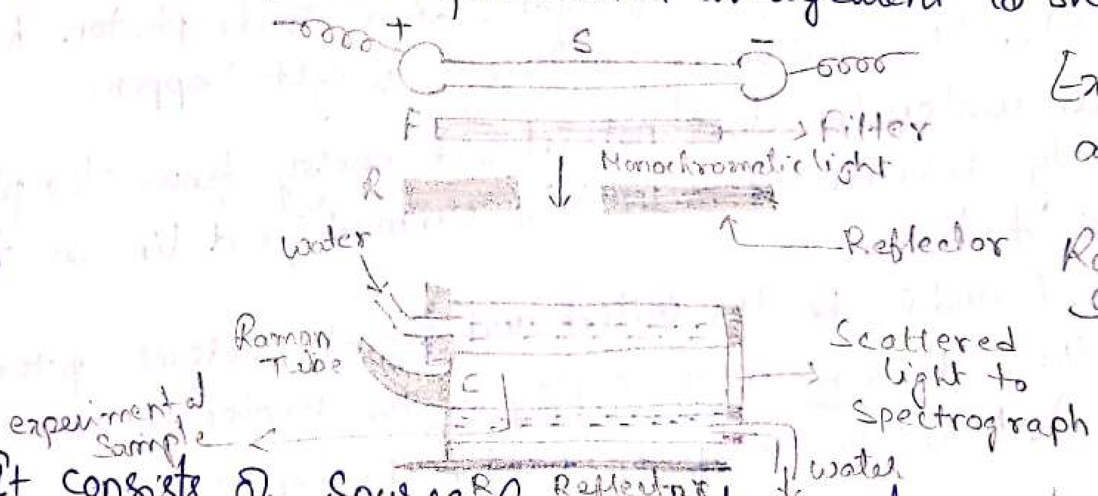
(4) anti stokes are weaker than stokes lines

(5) The rotational and vibrational state of molecules changed due to scattering of light by it.

19 (6) Raman effect is purely a molecular phenomenon.

→ Experimental Study of Raman effect and its applications:-

Here the source of light should be very strong otherwise the Raman lines will be of very low intensity (Now Lasers). The experimental arrangement is shown in fig.



Experimental arrangement of a Raman Spectrum.

It consists of Source, Raman tube and spectrograph. In the fig S is mercury arc lamp i-e a source of light. The light is passed through a filter F to obtain a monochromatic beam i-e light of a single freq. The light is then allowed to pass through an opening in a metallic reflector which then fall on Raman tube.

The Raman tube consists of a glass tube about 1 & 2 cm in diameter and 10 to 15 cm long. one end of the tube has a flat glass surface through which the scattered light emerges.

The other end is drawn out into horn-shaped and blackened outside to provide a black background. The tube is surrounded by a water jacket in which cold water is circulated to prevent overheating the sample in Raman tube due to proximity of exciting arc. The experimental sample is placed inside Raman tube. The scattered beam emerges from the flat end of the Raman tube and is examined by means of a spectrograph.

(20)

Quantum Theory of Raman effect:-

Based on quantum theory prof Smekal 1929 explained the Raman effect. According to quantum theory the source of light emits photon & light quanta of energy $h\nu_0$. where ν_0 is freq of light. when such photon hits a molecule the following 3 things might happen.

(1) The molecules does not absorb energy from the photon merely deviate it. In this unmodified line in the scattered beam.

(2) If E_1 and E_2 be the initial and final energies of the molecule, then decrease in the energy of the molecule will be (E_1-E_2). Now the energy of photon becomes

$h\nu_0 + (E_1-E_2)$. Thus the freq of scattered radiation is given by

$$\nu_{a.s} = \frac{h\nu_0 + (E_1-E_2)}{h} = \nu_0 + \frac{(E_1-E_2)}{h} \longrightarrow (1)$$

$\nu_{a.s}$ ↓ antistokes line

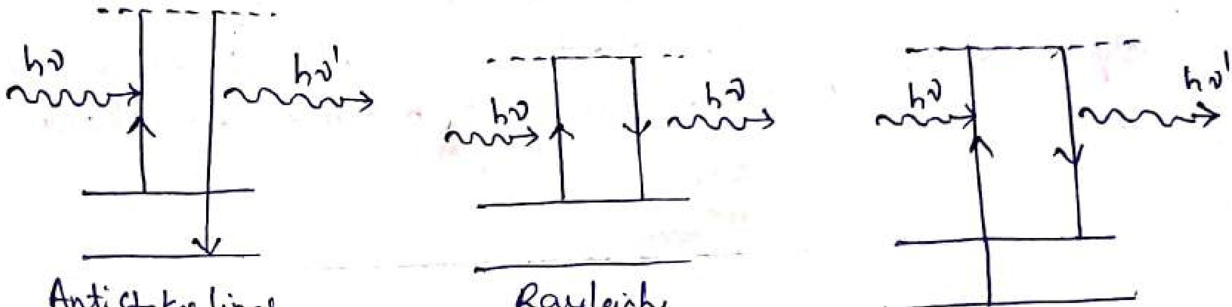
This shows that the freq of the scattered radiation is increased. This corresponds to the freq of antistokes line.

(3) The molecule may absorb some energy from the photon. Let the energy of molecule increases from E_2 to E_1. Now the photon's energy becomes $h\nu_0 - (E_1 - E_2)$. Thus, the freq of the scattered radiation is given by

$$\nu_s = \frac{h\nu_0 - (E_1 - E_2)}{h} = \nu_0 - \frac{(E_1 - E_2)}{h} \longrightarrow (2)$$

This shows that the freq of scattered radiation is decreased. This corresponds to the frequency of Stoke line.

(2) The three different situations are shown in fig



Anti stokes lines $\lambda' < \lambda$ | Rayleigh $\lambda' = \lambda$ | Stokes lines $\lambda' > \lambda$

For transitions b/w different energy levels of the molecules we get lines of different frequencies in the spectrum of scattered light. The lines with freq $\nu_{a.s}$ and ν_s respt the anti-Stokes lines and Stokes lines in Raman Spectrum of the molecule.

→ Applications of Raman effect:-

1) To study the molecular structure of crystals and compounds
2) To know the no of atoms in a molecule
3) To study the composition of plastics, mixtures.
4) To decide about single, double & triple bond.
5) To study the spin and statistics of nuclei
6) To study the binding forces b/w atoms, group of atoms in crystals
7) To study the vibrational and rotational energy levels of homonuclear Eg:- nitrogen, oxygen.

Scanned with CamScanner

V Sem / paper VI / UNIT: II

2 Matter waves & Uncertainty Principle.

particle :- It has mass, it is located at some definite point, it can move from one place to another, it gives energy when slowed down & stopped. Thus the particle is specified by 1) mass(m) 2) velocity(V) 3) momentum p. 4) energy E

Wave :-
A wave is spread out over a relatively large region of space, it can't be said to be located just here and there. Actually a wave is nothing but rather a spread out disturbance. A wave specified by its 1) frequency 2) wavelength 3) phase of wave velocity 4) amplitude and intensity.

Radiation is a wave which is spread out over space and also a particle which is localised at a point in space. Radiation sometimes behaves as a wave and other times as a particle. It has a wave-particle dualism.
Radiations including visible light, infrared, ultraviolet, X-rays etc.

De Broglie's Hypothesis of wave length of Matter waves:-

Louis de Broglie in 1924 extented wave particle parallelism of optics to all the fundamental entities of physics such as electrons, protons, neutrons, atoms and molecules etc. A moving particle has always got a wave associated with it and the particle is controlled by the wave.

According to de-Broglie's hypothesis, a moving particle is associated with a wave which is known as de-Broglie wave. The wavelength of the matter wave is given by $\lambda = \frac{h}{mv} = h/p$ m - mass, v - velocity, p - momentum

Considering the planks theory of radiation the energy of a photon is given by

$$E = h\nu = \frac{hc}{\lambda} \longrightarrow (1)$$

c - velocity of light
λ - wavelength

According to Einstein energy-mass relation

$$E = mc^2 \longrightarrow (2)$$

from eq (1) & (2) we get

2

$$mc^2 = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{mc^2} \Rightarrow \lambda = \frac{h}{mc} \longrightarrow (3)$$

where $mc = p$ (momentum associated with photon)

Consider a material particle of mass m and moving with a velocity v i.e momentum mv then the wavelength associated with this particle is given by

$$\boxed{\lambda = \frac{h}{mv} = \frac{h}{p}} \longrightarrow (4)$$

a) If E is the kinetic energy of the material particle then

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$$

&

$$p = \sqrt{(2mE)}$$

& de-Broglie wavelength $\boxed{\lambda = \frac{h}{\sqrt{(2mE)}}} \longrightarrow (5)$

b) When a charged particle carrying a charge q is accelerated by a potential difference V volts, then its K.E, E is given by

$$\boxed{E = qV}$$

Hence the de Broglie wavelength associated with this particle is given by

$$\boxed{\lambda = \frac{h}{\sqrt{2mqV}}} \longrightarrow (6)$$

c) When a material particle is in thermal equilibrium at a temp T, then

$$E = \frac{3}{2} KT$$

where K - Boltzmann's const $= 1.38 \times 10^{-23}$ J/K

So, de-Broglie wavelength of a material particle at temp T is given by

3

$$\lambda = \frac{h}{\sqrt{(2m(3/2 \cdot KT))}} \quad \& \quad \boxed{\lambda = \frac{h}{\sqrt{(3m_1 KT)}}} \longrightarrow (7)$$

→ properties of Matter waves :-

1) lighter in the particle, greater is the wavelength
2) smaller the velocity, greater is the wavelength
3) when $v=0$, then $\lambda = \infty$ – wave indeterminate
   if $v = \infty$ then $\lambda = 0$ – matter waves generated by motion of particles.
4) velocity of matter wave depends on velocity of matter particle
5) velocity of matter wave is greater than velocity of light
6) wave is large (strong) – chance of finding particle
   wave is small (weak) – very small chance of finding particle

Wave velocity and Group velocity :-

Wave Velocity & phase velocity :-

When a monochromatic wave i-e wave of single frequency and wavelength travels through a medium, its velocity of advancement in the medium is called as wave velocity.
Consider a wave whose displacement y is expressed as

$$y = a\sin(\omega t - kx)$$

a- is amplitude, ω - angular freq ($2\pi n$), $K = 2\pi/\lambda$ - const
The ratio of angular frequency ω to the propagation const K is defined as wave velocity. This is expressed by v_p. Hence

A

$$\boxed{v_p = \frac{\omega}{K}} \longrightarrow (1)$$

For the wave $(\omega t - kx)$ is the phase of wave motion. For the planes of const phase (wave fronts), we have

$$\omega t - kx = \text{const} \longrightarrow (2)$$

diff eq (2) w.r.t t

$$\omega - k\frac{dx}{dt} = 0$$

$$\boxed{\frac{dx}{dt} = \frac{\omega}{K} = v_p} \longrightarrow (3)$$

Thus, wave velocity is the velocity with which the planes of const phase advance through the medium. Due to this reason wave velocity also called phase velocity.

Group velocity :-

Consider a pulses rather than monochromatic wave. The pulse consists of a no of waves slightly differing in freq from one another. The superposition of such waves is known as wave group (wave packet). When such group travels in the medium, the phase velocities of diff components are different.

And the velocity ~~is~~ has maximum amplitude of the group advances. This is called group velocity. Thus, the group velocity is the velocity with which the energy in the group is transmitted.

Relation b/w Group & phase velocity :-

From eq ① $\frac{\omega}{K} = v_p \Rightarrow \omega = v_p K$.

5

& $d\omega = dv_p K + v_p dK$

$$\frac{d\omega}{dK} = v_p + K\frac{dv_p}{dK} \longrightarrow (4)$$

we know that $\boxed{v_g = \frac{d\omega}{dK'} = \frac{\omega - \omega'}{K - K'}} \longrightarrow (5)$ group velocity

From eq ④ & ⑤ we get

$$v_g = v_p + K\frac{dv_p}{dK}$$

& $$v_g = v_p + K\frac{dv_p}{d\lambda} \times \frac{d\lambda}{dK} \longrightarrow (6)$$

Since $K = \frac{2\pi}{\lambda}$ ω wavelength hence $\frac{d\lambda}{dK} = -\frac{2\pi}{K^2}$

Now eq ⑥ becomes

$$\boxed{v_g = v_p - \lambda\frac{dv_p}{d\lambda}} \longrightarrow (7)$$

eq ⑦ represents the Relation b/w group velocity v_g and phase velocity v_p.

→ **Davisson and Germer's Electron Diffraction Experiment :-**

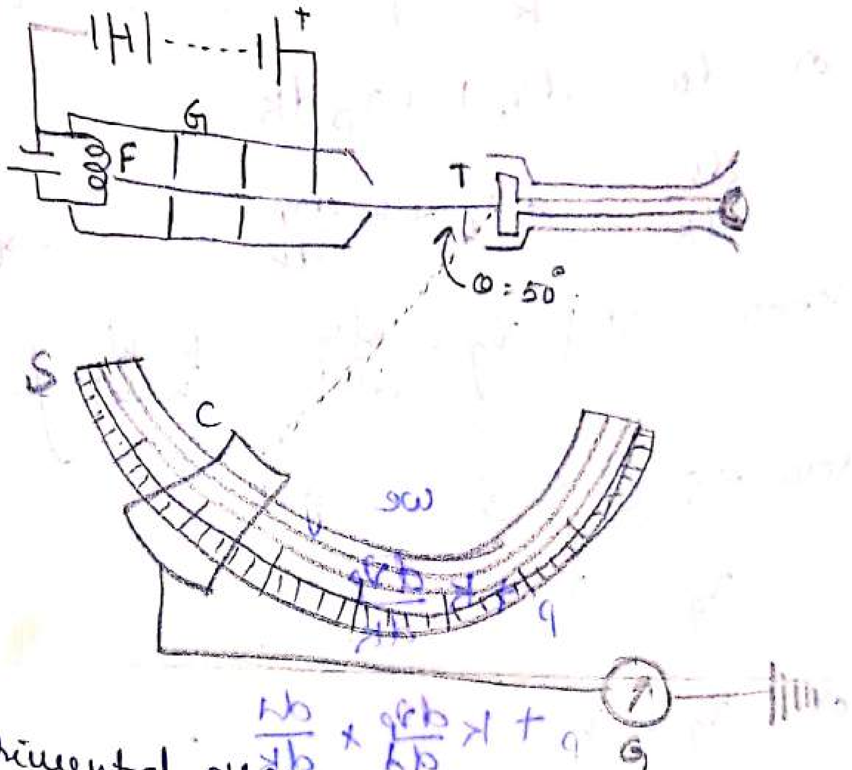
→

Experiment:-

scale electron flow

The first experimental evidence of matter wave was given by two American physicists Davisson and Germer in 1927. As nickel target was subjected to such heat treatment the electrons are diffracted like X-rays i.e they behave like waves under certain conditions.

6



The experimental arrangement is shown in fig. The apparatus consists of an electron Gun G, where the electrons are produced and obtained in a fine pencil of electronic beam of known velocity. The electron gun consists of a tungsten filament F heated to dull red so that electrons are emitted due to thermionic emission.

Now electrons are accelerated in the electric field of known potential difference, and are collimated by suitable slits to obtain a fine beam. The beam of the electrons is directed to fall on a large single crystal of ~~nik~~ nickel, known as target T. The electrons acting as the waves are diffracted in different directions.

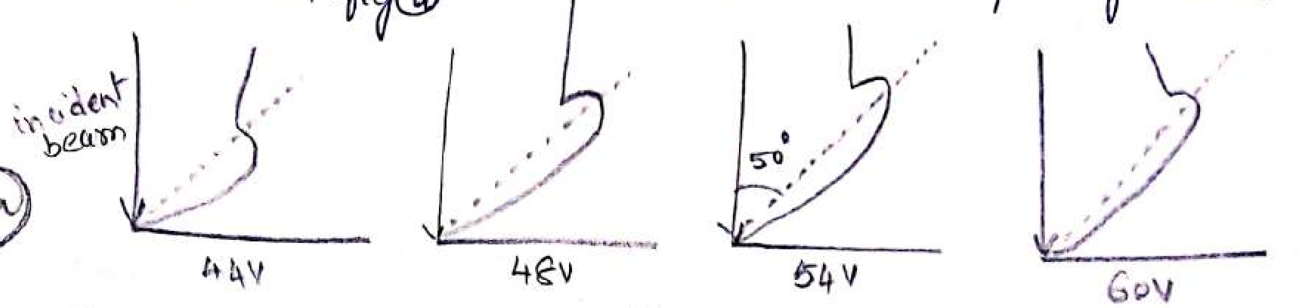
The angular distribution is measured by an electron detector which is connected to a galvanometer. Faraday cylinder C can move on a circular graduated scale S between 29° to 90° to receive the reflected electrons. C is consists of two walls which are insulated from each other.

Firstly the accelerating potential V is given a low value and the crystal is set at any arbitrary azimuth θ. Now C is moved to various positions ~~and~~ on the Scale S and galvanometer current is noted for each position.

7

A graph is then plotted b/w galvanometer current against angle θ b/w incident beam and beam entering the cylinder. The observations are repeated for different accelerating potentials. The corresponding curves are shown in fig (a)

(a)



It is observed that a 'bump' begins to appear in the curve for 44V electrons.

1) increasing potential the bump moves upwards
2) The bump becomes most prominent in the curve for 54V electrons at $\theta = 50°$
3) At higher potential the bumps gradually disappear.

According to De Broglie the wave length associated with electron accelerated through a potential V is given by

$$\lambda = \frac{12.26}{\sqrt{V}} A°$$

Hence the wavelength associated with 54 volts electrons is

$$\lambda = \frac{12.26}{\sqrt{54}} = 1.67 A°.$$

→ Davisson and Germer's Electron Diffraction Experiment.

8. Uncertainty principle

→ Heisenberg's uncertainty principle :-

In 1927 Heisenberg proposed a principle which is a direct consequence of the dual nature of matter known as uncertainty principle.

8 In classical mechanics moving particle at any instant has a fixed position in space and a definite momentum which can be determined.

In wave mechanics, the particle is described in terms of a wave packet. when wave packet is small it is possible to find position, But when wavepacket is large the velocity can be fixed but not position. Then there is certainty of momentum and uncertainty in position.

According to Heisenberg uncertainty principle, it is impossible to measure both the position and momentum of a particle.

Quantatively this principle states that the order of magnitude of the uncertainties in the knowledge of two variables must be at least planck's const h. Consider the pair of physical variable as position and momentum we have

$$\Delta p \, \Delta x \approx h \longrightarrow (1)$$

Δp – uncertainty in determining the momentum
Δx – " " " " position.

We have llly

$$\Delta E \, \Delta t \approx h \longrightarrow (2)$$

$$\Delta J \, \Delta \theta \approx h \longrightarrow (3)$$

ΔE and Δt are uncertainties in determining the energy and time. While ΔJ and $\Delta \theta$ are " angular momentum and angle.

The exact statement of uncertainty principle is as follows:-

The product of uncertainties in determining the position and momentum of the particle can never be smaller than the order of $h/4\pi$, so we have

$$\Delta p \Delta x \geq \frac{h}{4\pi} \;;\; \Delta E \cdot \Delta t \geq \frac{h}{4\pi} \;,\; \Delta J \cdot \Delta \theta \geq \frac{h}{4\pi}$$

Time - Energy uncertainity principle :-

The time - energy uncertainty relation can be obtained by considering a free particle with rest mass m_0 moving along x-direction with velocity v_x. The K.E is given by

$$E = \frac{1}{2} m_0 v_x^2 = \frac{P_x^2}{2m_0} \longrightarrow (1)$$

If ΔP_x and ΔE be the uncertainties in momentum and energy then diff eq (1) we have

$$\Delta E = \frac{2P_x \cdot \Delta P_x}{2m_0}$$

$$P_x \Delta P_x = m_0 \Delta E$$

$$\Delta P_x = \frac{m_0}{P_x} \Delta E = \frac{1}{V_x} \Delta E \longrightarrow (2)$$

Let the uncertainty in the time interval for measurement at point x be Δt then uncertainity Δx in position is

$$\Delta x = V_x \cdot \Delta t \longrightarrow (3)$$

from eq (2) and (3) we get

Scanned with CamScanner

$$\Delta x \cdot \Delta p_x = \Delta t \cdot \Delta E \longrightarrow (4)$$

~~lah~~ we know that $\Delta x \cdot \Delta p_x \geq \frac{h}{4\pi}$

10

$$\Delta t \cdot \Delta E \geq \frac{h}{4\pi} \longrightarrow (5)$$

according to Bohr's orbit concept $\Delta E = 0$, Now according to uncertainty relation Δt must be infinite. This shows that the energy states of the atom must have infinite life time. And the life time of atom in excited state is of the order of 10^{-8} sec. Thus, concept of Bohr orbits violates the uncertainty principle.

Experimental verification:-

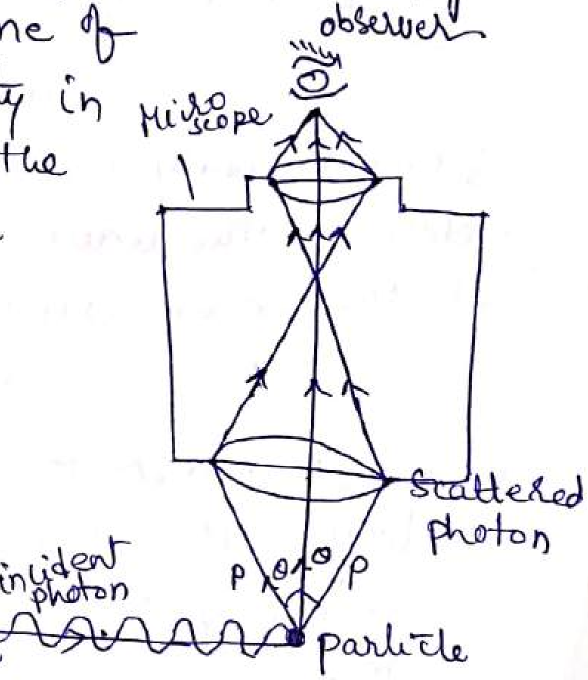
1) Determination of the position of a particle by microscope:-

Consider the case of the measurement of the position of particle say electron in the field of micro scope. The resolving power i.e the smallest distance between the two points that can be just resolved by the microscope is given by

11

$$\Delta x \approx \frac{\lambda}{2\sin\theta} \longrightarrow (1)$$

Where λ is the wavelength of light used. θ fig (1) the semivertical angle of the cone of light and Δx, the uncertainity in determining the position of the particle. In order to observe electron ~~it~~ is one photon must strike the electron and scatter inside the microscope. When a photon of initial momentum $p = h/\lambda$ after scattering enters the field of view of microscope it may be anywhere within the angle 2θ. Thus its x component of momentum i.e p_x may lie b/w $p\sin\theta$ and $-p\sin\theta$. As the momentum is ~~cos~~ conserved in the collision, the uncertainity in the x component of momentum is given by



$$\Delta p_x = p\sin\theta - (-p\sin\theta)$$

$$= 2p\sin\theta$$

$$= \frac{2h}{\lambda}\sin\theta \longrightarrow (2)$$

From eq ① and ② we have

$$\Delta x \cdot \Delta p_x \approx \frac{\lambda}{2\sin\theta} \times \frac{2h}{\lambda}\sin\theta.$$

12

$$\Delta x \cdot \Delta p_x \approx h. \longrightarrow ③$$

This shows that the product of uncertainties in position and momentum is of the order of planck's const.

(Two methods ①, ② :- Diffraction by a single slit) :-

→ **Complementary principle of Bohr :-**

This is most important consequence of uncertainty principle is that it is impossible to determine the wave and/or particle properties exactly at the same time.

According to this CPB the wave and particle aspects of matter and light are complementary rather than contradictory. i.e both aspects are necessary to have a complete picture of the same system.

For eg :- consider an experimental arrangement in which light is diffracted by a double slit and is detected on a screen that consists of many adjacent photoelectric cells. The photoelectric cells responds to photons and if we plot the number of photons each cell counts in a certain period of time against the location of the cell, a wave like pattern is obtained.

It ~~fda~~ follows that only expt. which can be devised displays either the particle like characteristics of wave like characteristics of the

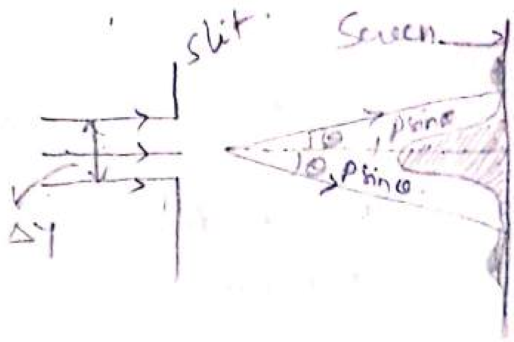
System, moreover the wave and particle pictures give complementary description of the same system.

13

→ Experimental verification (2nd Method) :-

(Diffraction by a single slit) :-

Suppose a narrow beam of electrons passes through a single narrow slit and produces a diffraction pattern on the screen in fig ① The first minimum of pattern is obtained by putting $n=1$ in the equation, i.e



$$d \sin\theta = n\lambda.$$

Hence $$\Delta y \sin\theta = \lambda \quad \text{—— ①}$$

where Δy is the width of the slit and θ is the angle of deviation corresponding to first minimum.

In producing the diffraction pattern on the screen all the electrons have passed through the slit but we can't say definitely at what place of the slit. The uncertainity in determining the position of the electron is equal to the width Δy of the slit. eq ① we have $$\Delta y = \frac{\lambda}{\sin\theta} \quad \text{—— ②}$$

Initially the electrons are moving along the x-axis and hence they have no component of momentum along y axis. After diffraction at the slit they are deviated from their initial path to form the pattern and have component $p \sin\theta$.

As y component of momentum ~~all~~ may lie anywhere b/w $p\sin\theta$ and $-p\sin\theta$, uncertainity in y component of momentum is

$$\Delta p_y = 2p\sin\theta = 2\frac{h}{\lambda}\sin\theta \quad \left(\because p = \frac{h}{\lambda}\right) \longrightarrow (3)$$

Hence from eq (2) & (3)

14 $$\Delta y \cdot \Delta p_y \approx \frac{\lambda}{\sin\theta} \times \frac{2h\sin\theta}{\lambda} \approx 2h.$$

&

$$\Delta y \cdot \Delta p_y \approx h \longrightarrow (4)$$

This relation shows that the product of uncertainities in position and momentum is of the order of planck's const.

V Sem | UNIT-III 3. Quantum (wave) Mechanics.
Paper - VI

(1)

Introduction :-

Bohr's theory Sucessful in explaining hydrogen like spectrum. But fails to explain - fine structure of spectral lines,
- distribution of electron in atoms,

classical mechanics provides the correct explanation for the behaviour of macroscopic system (position, velocity, momentum acceleration etc).
CM fails to explain microscopic system of particles.
Bohr's model failed to explained the behaviour of atomic system.
According to de Broglie theory - a material particle is associated with matter wave, and wave mechanics & quantum mechanics was developed in 1926 by Schroedinger.
Schroedinger described the amplitude of matter waves by a complex quantity $\psi(x, y, z, t)$ known as wavefunction & the State of the System. It describes the particular dynamical system under observation.

→ Schroedinger time independent wave equation :-

According to de Broglie theory, a particle of mass m is always associated with a wave whose wavelength is given by $\lambda = \frac{h}{mv}$. If the particle has wave properties, it is having some wave equation which describes particle.
Consider a System of Stationary waves and x, y, z be co-ordinates of the particle ψ. ψ is called as wave function and is assumed that ψ is finite

Single valued and periodic function. The classical differential equation of a wave motion is given by

$$\frac{\partial^2 \psi}{\partial t^2} = v^2\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right) = v^2 \nabla^2 \psi \longrightarrow (1)$$

Where $\nabla^2 = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}$ (∇^2 being Laplacian operator ~~equa~~)

v – the wave Velocity.

The Solution for eq (1) is given by

$$\psi = \psi_0 \sin \omega t = \psi_0 \sin 2\pi \mathbf{\nu} t \longrightarrow (2)$$

Where ν is frequency of the Stationary wave associated with the particle.

Differentiating eq (2) twice, we get

$$\frac{\partial \psi}{\partial t} = \psi_0 (2\pi \mathbf{\nu}) \cos 2\pi \mathbf{\nu} t \qquad \left\{ \because \nu = v/\lambda \right\}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\psi_0 (2\pi \mathbf{\nu})^2 \sin 2\pi \mathbf{\nu} t$$

& $$\frac{\partial^2 \psi}{\partial t^2} = -4\pi^2 \mathbf{\nu}^2 \psi = -\frac{4\pi^2 v^2}{\lambda^2}\psi \longrightarrow (3)$$

Substitute the value of $\frac{\partial^2 \psi}{\partial t^2}$ from eq (3) in eq (1) we get

$$v^2 \nabla^2 \psi = \frac{-4\pi^2 v^2}{\lambda^2}\psi \quad \& \quad \nabla^2 \psi + \frac{4\pi^2}{\lambda^2}\psi = 0 \longrightarrow (4)$$

Now from de-Broglie relation

$$\lambda = \frac{h}{mv} \quad \therefore \quad \nabla^2 \psi + \frac{4\pi^2}{h^2} \cdot m^2 v^2 \psi = 0 \longrightarrow (5)$$

If E and V be the total and potential energies of the particle, then its K.E $\frac{1}{2}mv^2$ is given by

3

$$\frac{1}{2}mv^2 = E - V$$

$$m^2v^2 = 2m(E-V) \longrightarrow (6)$$

from eq (5) and (6) we have

$$\nabla^2\psi + \frac{4\pi^2}{h^2}\times 2m(E-V)\psi = 0.$$

$$\therefore \boxed{\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0} \longrightarrow (7)$$

Eq (7) is known as Schroedinger time independent wave equation.

Substituting $\hbar = \frac{h}{2\pi}$ in eq (7) the Schroedinger's wave equation can be written as

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0} \longrightarrow (8)$$

No Need

Eq. 8 can also be expressed in the following way

$$\left(\frac{\hbar^2}{2m}\right)\nabla^2\psi + (E-V)\psi = 0$$

$$\frac{\hbar^2}{2m}\nabla^2\psi - V\psi = -E\psi$$

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi = E\psi$$

$$\boxed{\hat{H}\psi = E\psi} \longrightarrow (9)$$

Where $\hat{H} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)$ ~~ψ and H is~~ known as Hamilton ~~can operator~~. For a free particle $V=0$ hence the Schroedinger wave equation for a free particle can be expressed as

$$\boxed{\nabla^2\psi + \frac{2mE}{\hbar^2}\psi} = 0 \longrightarrow (10)$$

(4)

→ physical Significance of wavefunction ψ

In any electromagnetic wave system if A is the amplitude of the wave then the energy density, i-e energy per unit volume is equal to A^2, so that the no of photons per unit volume, i-e photon density is equal to $A^2/h\nu$ & the photon density is proportional to A^2 as $h\nu$ is const.

If ψ is the amplitude of matter waves at any point in space, then the particle density at that point may be taken as proportional to ψ^2.

Thus ψ^2 is a measure of particle density; when this is multiplied by the charge of the particle the charge density is obtained. In this way ψ^2 is a measure of charge density.

Max Born suggested a new idea about the physical significance of ψ which is generally $\psi\psi^* = |\psi|^2$ gives the probability of finding the particle in the state ψ. i.e ψ^2 is a measure of probability density.

The probability of finding a particle in volume $d\tau = dx\,dy\,dz$ is given by $|\psi|^2 dx\,dy\,dz$. For the total probability of finding the particle somewhere is unity i.e particle is certainly to be found somewhere in space

$$\iiint |\psi|^2 dx\,dy\,dz = 1.$$

ψ satisfying above requirement is said to be normalized.

→ **Wave function properties :-**

The wave function ψ must fulfil the following requirements.

1. It must be finite every where.
2. It must be single valued
3. It must be continuous.

If ψ is infinite at particular point, then it would mean an infinitely large propability of finding the particle at that point. This is not possible. Hence ψ must have a finite & zero value at any point.

2. Let us consider that ψ has more than one value at any point. It means that there is more than one value of probability of finding the particle at that point. This is obviously ~~bad~~ ridiculous.

3. For Schroedinger equation $\frac{d^2\psi}{dx^2}$ must be finite everywhere. This is possible only when $d\psi/dx$ has no discontinuity across a boundary.

Important points:-

1) Since a physical system must exist somewhere, the probability of its finding must be 1. i.e

$$\int_{-\infty}^{+\infty} |\psi|^2 \, dx\,dy\,dz = 1.$$

6

This is known as normalization condition.

2) The probability of finding the system between x_1 and x_2 is given by

$$P = \int_{x_1}^{x_2} |\psi|^2 dx.$$

3) The probability per unit volume of a system being in the state ψ is called as probability density.

$\rho = |\psi|^2 dx.$

The total probability should be 1. Therefore

$$\int_V \rho\, dv = \int |\psi|^2 dv = 1.$$

→ Operators:-

An operator $\hat{O}$ is a mathematically operation which may be applied to a function $f(x)$ that changes the function to another function $g(x)$. This can be represented as

$$\hat{O} f(x) = g(x). \longrightarrow (1)$$

So an operator is a rule by means of which from a given function, we can find another function.

For example $\frac{d}{dx}(x^3) = 3x^2 \longrightarrow (2)$

i.e. When the differential operator (d/dx) operates on the function x^3, the function x^3 is changed to another function $(3x^2)$

It is convenient to regard the expression $\frac{d}{dx} f(x)$ as consisting of two constitutents

7 the operator (d/dx) and the operent $f(x)$. Now

$$\frac{d}{dx} f(x) = f'(x). \longrightarrow (3)$$

i-e When the operator (d/dx) operates on the function $f(x)$ it gives the first derivative of $f(x)$ obtained by the rules of differential calculus.

Operator Algebra:-

The operators used in quantum mechanics are linear operators. The operator algebra of linear operators is as follows.

(1) Sum of the operators:-

If A and B are two linear operators then the sum $\hat{C} = \hat{A} + \hat{B}$ is also a linear operator.

$$\hat{C}\hat{\psi} = (\hat{A}+\hat{B})\hat{\psi} = \hat{A}\hat{\psi} + \hat{B}\hat{\psi}$$

(2) Multiplication of operators:-

The multiplication of a linear operator $\hat{A}$ by a const a gives a linear operator $(a\hat{A})$ i-e

$$a\hat{A}\psi = (a\hat{A})\psi \longrightarrow (5)$$

(3) product of two operators:-

The product of two operators $(A \,\&\, B)$ is $\hat{A}.\hat{B}$ need not necessarily identical to the product $\hat{B}.\hat{A}$. For example

$$\left(x\frac{d}{dx}\right)f(x) = x\frac{df(x)}{dx}.$$

3

and $\left[\frac{d}{dx} x\right]\left[f(x)\right] = \frac{d}{dx}\left[x, f(x)\right]$

$$= f(x) + x\frac{d f(x)}{x}.$$

Hence $\quad x\frac{d}{dx} \neq \frac{d}{dx}x \longrightarrow (6)$

So in case of product of two operators, the way is most important.

8

(4) Commutator:-

The operator $(\hat{A}\hat{B} - \hat{B}\hat{A})$ is called the commutator of two operators $\hat{A}$ and $\hat{B}$. If $\hat{A}\hat{B} - \hat{B}\hat{A} = 0$, then the two operators $\hat{A}$ and $\hat{B}$ are called as commutating operators, Thus for commutating operators $\hat{A}\hat{B} = \hat{B}\hat{A} \longrightarrow (7)$

Different operators:-

Energy operator :- $\hat{E} = H = -\frac{\hbar^2}{2m}\nabla^2 + V = i\hbar\frac{\partial}{\partial t}.$

Momentum operator :- $\hat{p} = \frac{\hbar}{i}\nabla$

[redacted]

K.E operator :- $K.E = -\frac{\hbar^2}{2m}\nabla^2$

Velocity operator :- $\bar{v} = \frac{\hbar}{im}\nabla$

(Operators in Tabular form):-

→ Eigen values and Eigen Functions:-

A class function ψ which when operated by an operator $\hat{O}$ are merely multiplied by some const, say λ, i.e

$$\hat{O}\,\psi(x) = \lambda\,\psi(x)$$

then we say that the number λ is an eigen value of the operator $\hat{O}$ and the operand $\psi(x)$ is an eign function of $\hat{O}$.

for example since the operand $\sin 4x$ is well behaved function and if operated by an operator $\left(-\frac{d^2}{dx^2}\right)$ gives the following result.

$$\underbrace{-\frac{d^2}{dx^2}}_{\text{operator}} \sin 4x = \underbrace{16}_{\text{value}} \underbrace{\sin 4x}_{\text{function}}.$$

we say then the number 16 is an eigen value of the Operator $-d^2/dx^2$ and the operand $\sin 4x$ is an eigen function of the operator $-d^2/dx^2$

(9) In operator form, Schroedinger wave equation can be written as

$$H\psi = E\psi \longrightarrow (1)$$

Where $H = -\frac{\hbar^2}{2m}\nabla^2 + V$ and $E = i\hbar\frac{\partial}{\partial t}$

eq (1) can solve only for certain value of Engery E.

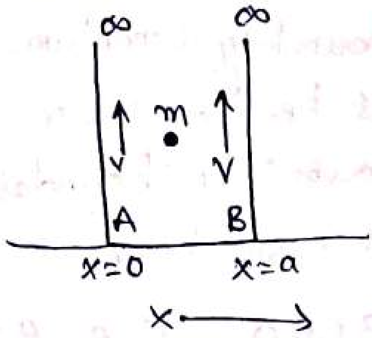
To solve schroedinger equation for a given system means to obtain a wavefunction ψ that obeys the equation and fulfils the requirements for an acceptable wave function

The values of Energy E_n for which Schroedinger steady state equation can be sloved are called eigen values and the corresponding wave functions ψ_n are called eigen functions.

→ Energy levels of a particle enclosed in one dimensional potential box of infinite height

Let us consider the case of a particle of mass m moving along x-axis b/w the two rigid walls A and B at $x=0$ and $x=a$. fig. The particle is free to move between the walls. The potential energy of the particle b/w two walls is const because no force is acting on the particle. The const Potential energy is taken as zero

(10)



When the particle strikes any one of the walls, it is reflected back as the walls are perfectly rigid. The force acting on the particle changes from zero to infinite.

we know that $|F| = \frac{\partial V}{\partial x}$ and we have $\Delta V \to \infty$ as $\Delta x \to 0$ Then

$\frac{\partial V}{\partial x}$ has infinite Value $|F|$.

The potential energy of the particle becomes infinite at the walls. Thus the potential function is defined in the following way.

$$V(x) = \infty \text{ for } x<0 \text{ and } x>a.$$

and $$V(x) = 0 \text{ for } 0 \le x \le a. \longrightarrow (1)$$

The Schroedinger wave equation for the particle is given by

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0$$

As $V=0$ b/w the walls, hence the equation has the following form

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi = 0 \longrightarrow (2)$$

Let $\frac{8\pi^2 m}{h^2}E = K^2$ then eq (2) takes the form

$$\frac{d^2\psi}{dx^2} + K^2\psi = 0 \longrightarrow (3)$$

The solution of eq (3) is of the form of $\sin Kx$ & $\cos Kx$. The general solution is given by

$$\psi(x) = A\sin kx + B\cos kx \longrightarrow (4)$$

A and B are Consts. The values of these constants can

Scanned with CamScanner

be obtained by applying boundary conditions. The particle cannot penetrate the walls hence $\psi = 0$ at $x = 0$ and $\psi = 0$ at $x = a$. These are the boundary conditions. Applying these conditions, we have

11

$$0 = A\sin 0 + B\cos 0. \quad \text{i.e } B = 0.$$

$$\psi(x) = A\sin kx.$$

Again $0 = A\sin ka$ $\quad\therefore \psi(x) = 0$ when $x = a$.

$A = 0$ & $\sin ka = 0$ but $A \neq 0$ because if $A = 0$ the entire function will be zero as $B = 0$.

$\sin ka = 0$ & $ka = n\pi \; (n = 0, 1, 2, 3 \ldots)$

& $$k = \frac{n\pi}{a} \longrightarrow (5)$$

Now the wave function becomes

$$\psi(x) = A\sin\frac{n\pi x}{a} \longrightarrow (6)$$

<u>Eigen Values</u>:-

From eq (5) $k^2 = \frac{n^2\pi^2}{a^2}$ and we have used k for $\frac{8\pi^2 mE}{\hbar}$

$$\therefore \frac{8\pi^2 mE_n}{h^2} = \frac{n^2\pi^2}{a^2}$$

{ using E_n for E generally }

$$\therefore \boxed{E_n = \frac{n h^2}{8ma^2} = n^2\frac{\pi^2\hbar^2}{2ma^2}} \longrightarrow (7)$$

$(n = 1, 2, 3 \ldots)$ and $(\hbar = h/2\pi)$

It is clear from expression that inside an infinitely deep potential well, the particle can have ~~de~~ discrete set of values of energy. The discreate values are given by

$E_1 = \frac{\pi^2\hbar^2}{2ma^2}$ for $n = 1$

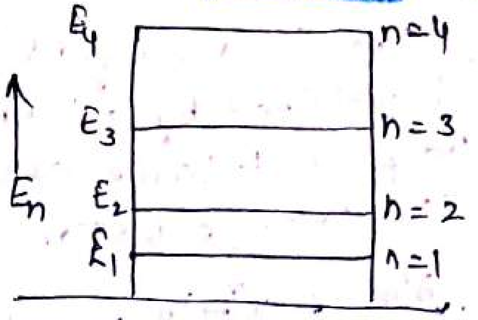
$E_2 = 4\frac{\pi^2\hbar^2}{2ma^2} = 4E_1$ for $n = 2$

$E_3 = 9\frac{\pi^2\hbar^2}{2ma^2} = 9E_1$ for $n = 3$

$E_4 = 16\frac{\pi^2\hbar^2}{2ma^2} = 16E_1$ for $n = 4$

The energy levels are shown in fig (2)
According to classical mechanics the particle may take continuous range of values between zero and infinity.

fig (2)



(12)

$$\boxed{\psi(x) = \sqrt{\left(\frac{2}{a}\right)} \sin \frac{n\pi x}{a}} \quad \text{(8)}$$

eq (8) gives the wave functions of the particle enclosed in infinitely deep potential well. The wave functions ψ_1, ψ_2 and ψ_3 corresponding to n=1, 2, and 3 are shown in fig (3)

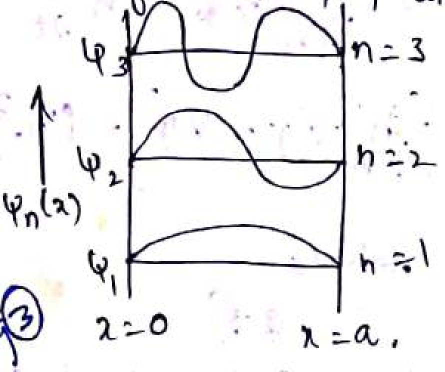


fig (3)

Between x=0 and x=a, the wavefunction has zero value, and these points are called nodes. for n=1, two nodes at x=0 and x=a. for ψ_1.
n=2, x=0, x=a/2, and x=a — ψ_2
n=3 x=0, x=a/3, x=2a/3, x=a — ψ_3

In general we have (n+1) nodes for wave function ψ_n.

→ Postulates of wave Mechanics:-

. The postulates of the quantum mechanics may be stated as follows:-

(X) for a wave function ψ to describe any physical system, the following boundary conditions must be satisfied.

(1) $\psi(x)$ as well as $d\psi(x)$ must be finite for all values of x
(2) $\psi(x)$ as well as $d\psi(x)/dx$ must be continuous for all values of x in the region except where the potential $V(x,y,z)$ is infinite.
(3) $\psi(x)$ as well as $d\psi(x)/dx$ must be single valued for all x in the region
(4) ~~In~~ the wavefunction $\psi(x)$ must satisfy the Born conditions

Schrodinger's time dependent wave equation

13

The Schrodinger's time dependent wave equation can be obtained by eliminating of E value in Schrodinger's time independent wave equation.

The wave equation is

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \nabla^2 \psi \longrightarrow (1)$$

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Its Solution is

$$\psi = \psi_0 e^{-i\omega t}$$

$$\frac{\partial \psi}{\partial t} = \psi_0 e^{-i\omega t}(-i\omega) \quad (\because \omega = 2\pi\nu)$$

$$\frac{\partial \psi}{\partial t} = \psi_0 e^{-i\omega t}(-i \cdot 2\pi\nu)$$

$$\frac{\partial \psi}{\partial t} = \psi_0 e^{-i\omega t}\left[-i 2\pi \frac{E}{h}\right] \quad \left\{\because E = h\nu,\ \nu = \frac{E}{h}\right\}$$

$$\frac{\partial \psi}{\partial t} = \psi\left[-i \frac{1}{\hbar} E\right] \quad \left\{\because \hbar = \frac{h}{2\pi},\ \psi = \psi_0 e^{-i\omega t}\right\}$$

$$\frac{\hbar}{i} \times \frac{\partial \psi}{\partial t} = -E\psi$$

Multiply & Divide with i on LHS

$$\frac{i\hbar \frac{\partial \psi}{\partial t}}{i^2} = -E\psi \quad [\because i^2 = -1]$$

$$i\hbar \frac{\partial \psi}{\partial t} = E\psi \longrightarrow (2)$$

Substitute eq (2) in Schrodinger's time independent equation

$$\nabla^2 \psi + \frac{2m}{\hbar^2}\left[E\psi - V\psi\right] = 0$$

$$\nabla^2 \psi + \frac{2m}{\hbar^2}\left[i\hbar \frac{\partial \psi}{\partial t} - V\psi\right] = 0$$

$$\nabla^2 \psi = -\frac{2m}{\hbar^2}\left[i\hbar \frac{\partial \psi}{\partial t} - V\psi\right]$$

$$\Rightarrow -\frac{\hbar^2}{2m}\nabla^2 \psi = i\hbar \frac{\partial \psi}{\partial t} - V\psi$$

$$\Rightarrow \left[-\frac{\hbar^2}{2m}\nabla^2 \psi + V\psi\right] = i\hbar \frac{\partial \psi}{\partial t}$$

~~where~~

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right] = i\hbar \frac{\partial \psi}{\partial t}$$

$$\Rightarrow \boxed{H\psi = E\psi}$$

Where

$$\Rightarrow H = -\frac{\hbar^2}{2m}\nabla^2 + V$$ — Hamiltonian operator

$$\Rightarrow E = i\hbar \frac{\partial}{\partial t}$$ — Energy operator.

V. 6. Crystal Structure.
7. Super conductivity

V Sem

paper - VI

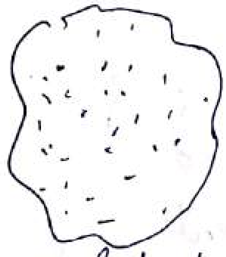
1

6) Crystal Structure:

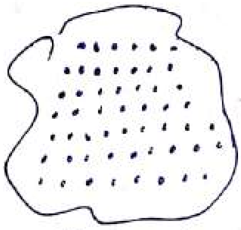
→ Amorphous and crystalline materials:-

Elements and their chemicals compounds are found to occur in three states. 1. Solids. 2. liquids. 3. gases. We know that some are strong, some are weak, some are good conductors of heat and electricity. Some are magnetic, some are non-magnetic etc.

If the atoms & molecules in a solid are arranged in some regular fashion, then it is known as crystalline. When the atoms & molecules in a solid are arranged in an irregular fashion then it is known as amorphous. fig shows the classification of solids according to atomic arrangement.



amorphous



crystalline.

1

A crystal is a solid composed of a periodic array of atoms i.e the study of solid - state physics aim to interpret the macroscopic properties in terms of properties of microscopic particles of which the solid is composed.

The study of the geometric form and other physical properties of crystalline solids by using X-rays electron beam and neutron beam etc is known as crystallography

→ Unit cell and crystal Lattice :-

Let us consider a two dimensional crystal in which the atoms are arranged as shown in fig ⓐ. Consider a parallelagram by any integral multiple of vectors ~~are~~ and b the whole crystal lattice may be obtained. In this way fundamental

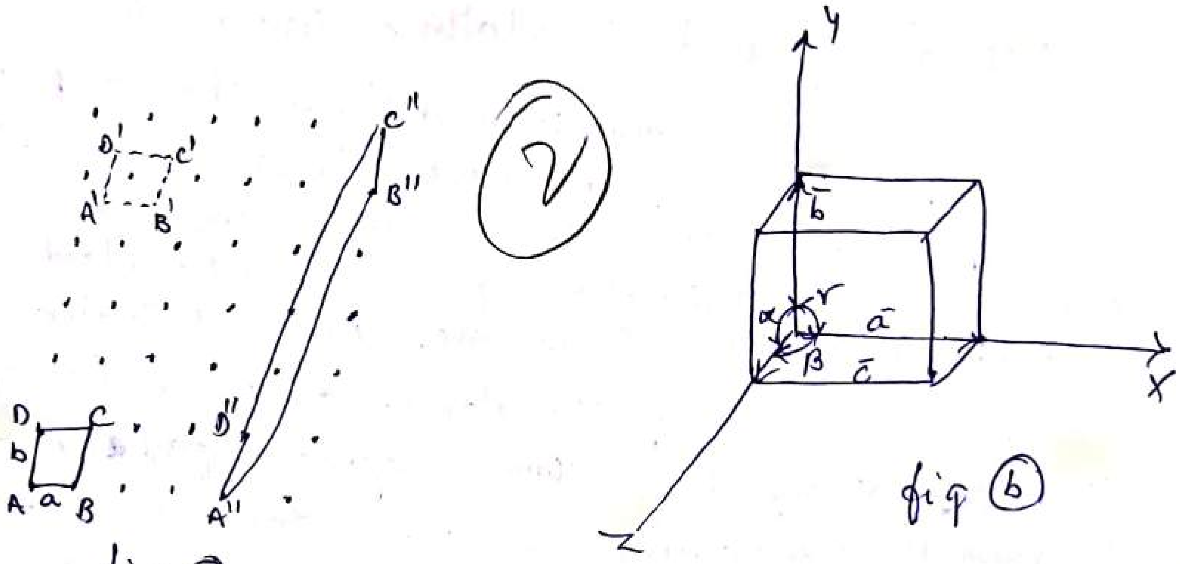


fig ⓐ

fig ⓑ

unit ABCD is called unit cell.

The unit cell is the smallest geometric fig the repetition of which gives the actual crystal structure.

It may also be defined as fundamental elementary pattern of minimum no of atoms, molecules or group of molecules which represents fully all the characteristics of the crystal.

A unit cell is constructed into a no of ways as $A'B'C'D'$ & $A''B''C''D''$ as shown in fig ⓐ. The unit cell should be schoosen in such a way that it converts the symmetry of crystal lattice and makes a mathematical calculation easy.

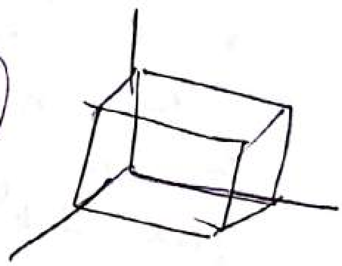
The unit cell is a parallelopiped formed by the basis vectors a, b, c as concurrent edges and ~~a~~ including angles α, β, γ b/w b and c, ~~c and a~~

c and a and a and b. as shown in fig (b). 2

Thus in general the unit cell ~~be~~ may be defined as that volume of a solid from which the entire crystal may be constructed by translational repetition in 3 dimensions.

A crystal lattice is a large assembly of points each point representing the repeating atoms & group of atoms in the crystal. Each point is called lattice point. The arrangement of points around a given lattice point is exactly similar to that of any lattice point around it,

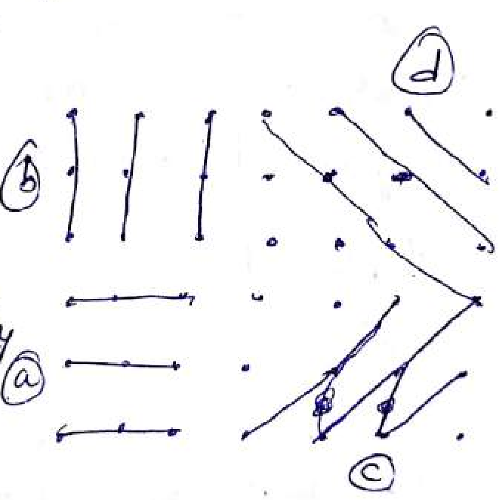
3



→ **Miller Indices:-**

The crystal lattice may be regarded as made up of aggregate of a set of parallel equidistant planes passing through the lattice points which are known as lattice planes. For given lattice the lattice planes can be chosen in a different no of ways for example (a), (b) (c) and (d) as shown in fig (1).

Miller evolved a method to designate a plane in a crystal by three numbers (h, k, l) known as Miller indices. This method is now universally employed.



The Miller indices are the three smallest possible integers which have the same ratio's as the reciprocals of the intercepts of the plane concerned on the three axes. According to Miller, the crystal planes could be described in terms of intercepts along three axes as shown in fig (1)

The intercepts on the 3 axes are (2, 3, 4) and the reciprocals are wholes no's and are denoted by h, k and l. These number are known as miller indices.

This Miller indices of a plane is defined as the reciprocals of intercepts which the plane makes with the axes when reduced to smallest number.

Rule's for finding Miller indices:- 4

1) First determine all the intercepts of plane on the 3 coordinates.
2) take the reciprocals of 3 intercepts
3) reduce reciprocals into whole numbers. This can be done by Multiplying each reciprocal by a no. obtain after taking LCM of denominators.

$$\begin{array}{r|rrr} 2 & 2 & 3 & 4 \\ \hline 2 & 1 & 3 & 2 \\ \hline & 1 & 3 & 1 \end{array}$$

Eg:- 1) intercepts are 2, 3, 4.

2) reciprocals are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

3) LCM of denominators 2, 3, 4 is 12. $2 \times 2 \times 1 \times 3 \times 1 = 12$

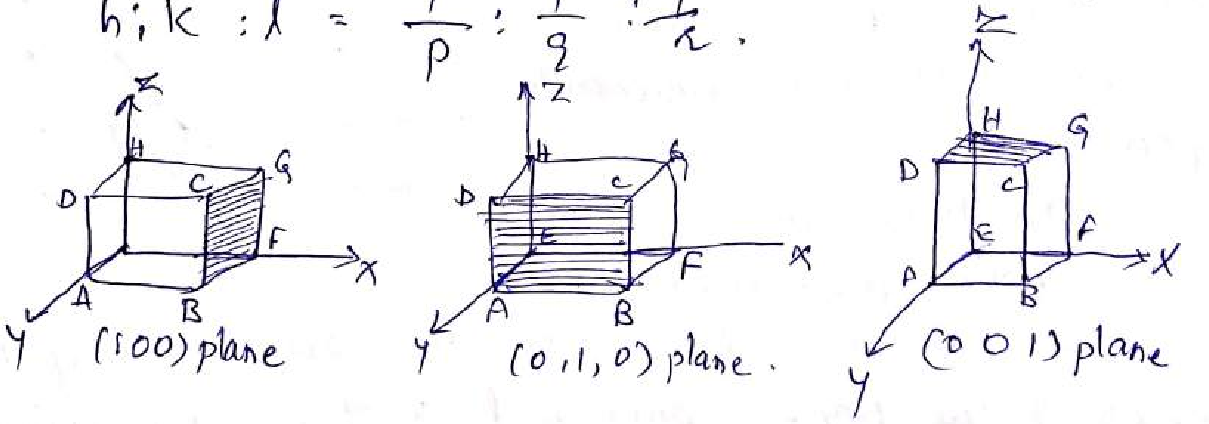
Hence multiply by 12, we have 6, 4, 3.

$$\frac{\cancel{12}^{6}}{\cancel{2}}, \frac{\cancel{12}^{4}}{\cancel{3}}, \frac{\cancel{12}^{3}}{\cancel{4}}.$$

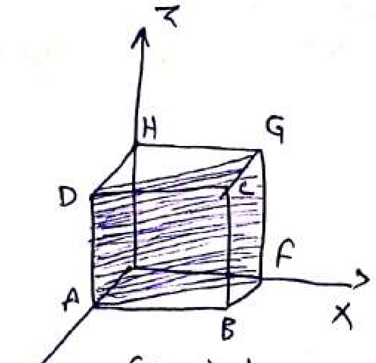
Thus miller indices of this plane is (6, 4, 3)

In general p, q, r are indices, reciprocals $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$

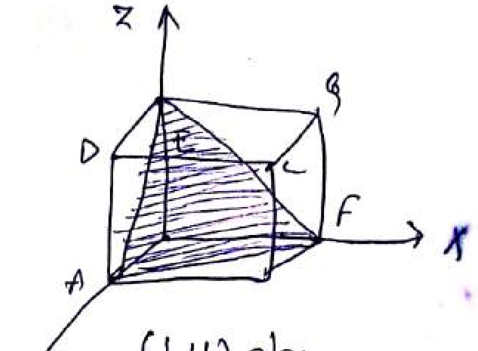
$$h : k : l = \frac{1}{p} : \frac{1}{q} : \frac{1}{r}.$$



(100) plane (0,1,0) plane. (0 0 1) plane



(110) plane



(1 11) plane

The no. h, k, l are called Miller indices of the given set of planes. The plane is specified as (h, k, l). The following are some planes in cubic crystal shown in above fig.

→ Reciprocal Lattice :- 5

P-VI
S-V

6. Crystal Structure:

Solid State physics ①
— S.O. pillai.
New age international publi

→ Reciprocal Lattice: (6)

physical properties of crystalline solids are different in different directions. W.H. Miller introduce the usage of a set of three integers known as Miller indices to represent a set of parallel planes.

Let us consider one such plane of Miller indices (h k l) within the unit cell of cubic crystal. The fractional co-ordinates of intercept of the plane (h k l) on the a, b, c axes are $1/h, 1/k, 1/l$ respty.

The equation of this plane may be written as

$$hx + ky + lz = 1 \longrightarrow ①$$

Refer to eq ① we can have two possible interpretations, one is h, k, l may be considered fixed and x, y, z are variable. In second case x, y, z may be considered fixed and h, k, l are variable. In first case a set of points lying on the plane and in the second case the same case represents a set of planes passing through a fixed point.

Bragg established a relationship b/w the angle of x-rays and their wavelength λ as

$$\lambda = 2d_{hkl} \sin\theta \longrightarrow ②$$

$$\lambda = 2d \sin\theta_{hkl} \longrightarrow ③$$

$$\lambda = 2d_{hkl} \sin\theta_{hkl} \longrightarrow ④$$

This equation indicates the diffraction is by the planes specified (h k l) at particular angle θ_{hkl} called Bragg angle of the plane (h k l).

eq ② helpful in relating the measured angle θ & the interplanar Spacing d_{hkl} in case of diffraction experiment. eq ③ is helpful in determining the unknown wavelength of X-ray by using measured angle θ as in Spectroscopic expts.

But to study the diffraction of x-ray by crystals it is also required to be known the interplanar spacing d as only they determine the reflection angles θ. Now if we draw normals to each set of parallel planes from a common origin and their lengths are made proportional to reciprocal of the interplanar spacings, the points at the end of these normals ~~from~~ a lattice array.

1

The distances in this array are reciprocal to distances in the crystal, the array is called the reciprocal lattice of the crystal.

The points in the reciprocal lattice are called reciprocal lattice points. These points in ~~are~~ three dimensional space ~~fro~~ form the reciprocal lattice space. This reciprocal lattice space is also called the K-space.

Construction:-

The general rules for constructing the reciprocal lattice may be formulated by the following steps.

① Fix up some point in the direct lattice as a common origin.

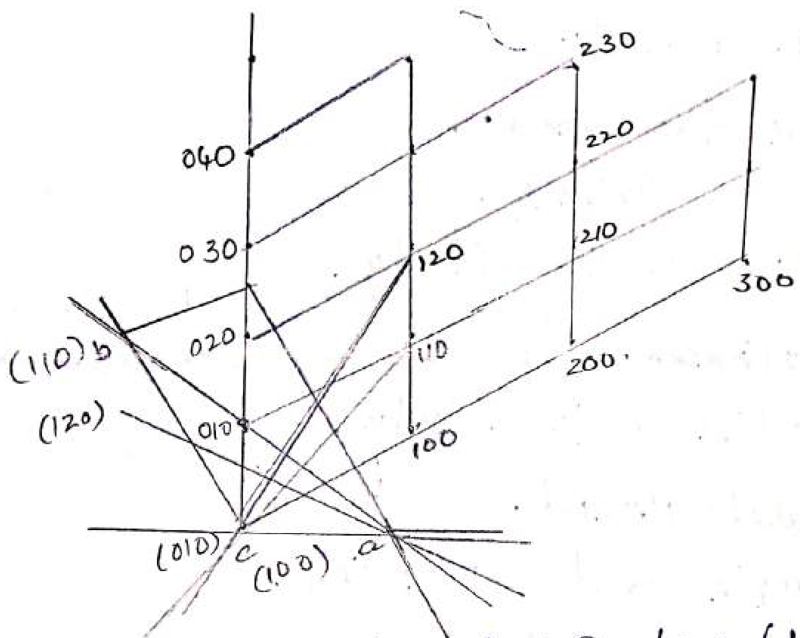
② From this common origin draw normals to each and every set of parallel planes in the direct lattice

③ Fix the length of each normal equal to the reciprocal of the inter planar spacing ($1/d_{hkl}$) of the set of parallel planes (hkl) it represents.

④ put a point at the end of each normal.

The collection of all these points in space is the reciprocal lattice space.

Let us consider as an example for construction of reciprocal lattice, consider planes belonging to a single zone. This makes the normal to all parallel planes belonging to the zone all to lie in the same plane. In fig ① the zone axis lies ‖er to the plane of the diagram and hence all normals to the parallel planes of the family of the zone will lie in the planes of the diagram.



Graphical construction of reciprocal lattice

8

fig ① shows the edge views of four (hkl) planes viz (100) (110) (120) and (010) all belonging to the (001) zone. It is clear that the normals to the family of planes (100) (200) (300) etc are parallel. It is also known that $d_{100} = 2d_{200} = 3d_{300} = 4d_{400}$ etc.

Hence $\sigma_{100} = K\left(\frac{1}{d_{100}}\right)$ $\sigma_{200} = 2\sigma_{100}$, $\sigma_{300} = 3\sigma_{100}$

Therefore the distance of the reciprocal lattice point representing the set of parallel planes (h00).

(May be continuation for essay)

A vector algebraic proof for 3D reciprocal lattice).

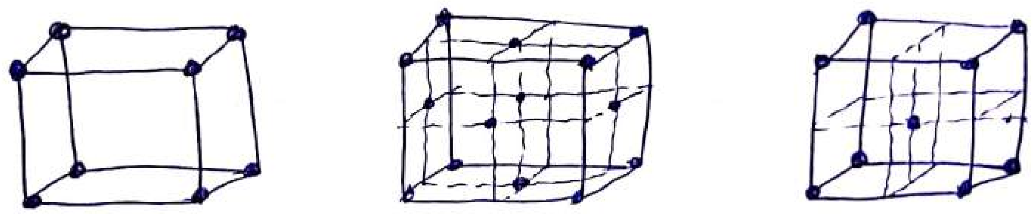
→ Types of Lattices :-

In 1848 Bravais showed that there are only fourteen ways of arranging points in space so that environment look the same from each point. The lattices are called the Bravais Lattices named after the discovery.

In the case of a cubic system, there are three Bravais Lattices each of which has the same collection of symmetry elements and the lattice points. The lattices are shown in fig and are refered as.

1) Simple cubic (Sc) (or) cubic p type lattice
2) Face centred cubic (Fcc) (or) cubic F Lattice
3) Body centred cubic (bcc) & cubic I lattice

9



Simple cubic (Sc) ⓐ   face centred cubic ⓑ (fcc)   body centred cubic (Bcc) ⓒ.

① Simple cubic (Sc) :- There is one Lattice point at each of eight corners of the unit cell. There is no lattice point inside the unit cell fig ⓐ

② face centred cubic (fcc) :- There is one Lattice point at each of the eight corners and one lattice point at the centres of each of the six faces of the cubic cell. Thus there is an extra point at the centre of each face as shown in fig ⓑ.

③ Body centred cubic (bcc) :- There is one Lattice point at each of the eight corners and one lattice point at the centre of each cell. So there is a lattice point at the centre of each unit cell fig ⓒ.

X-ray Diffraction: 26" ~~1~~

1,2,5,6,7,9,10,11, 12,13,16,17, 18

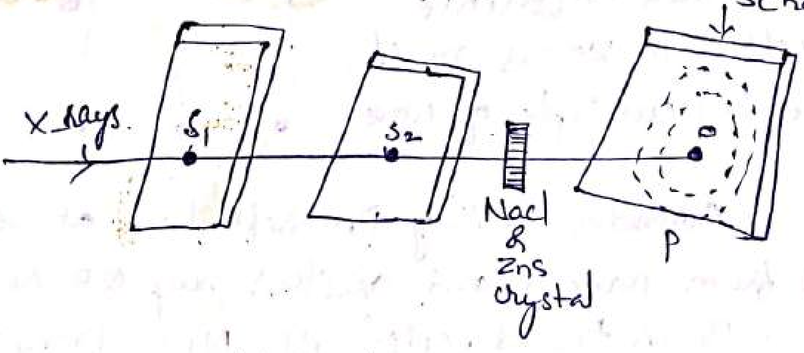
→ Diffraction of X-rays by crystals:-

The physical nature of X-rays are of two types. According to first theory:- X-rays were regarded as made of high speed particles like cathode rays but having a greater penetrating power. According to second theory X-rays were treated as electro magnetic waves of extremely high frequency. As after lot of experiments X-rays ~~the~~ diffraction pattern with a transmission grating is practically impossible.

In 1912 German ~~scientist~~ physicist Laue suggested that a crystal which consists of a 3D array of regularly spaced atoms can serve the purpose of a grating. The crystal acts as a space grating rather than a plane grating.

The experimental arrangement is shown fig.

10



X-rays obtained from X-ray tube are reduced to a narrow fine pencil by passing them through pin holed screens S_1 & S_2. The beam is now allowed to pass through a Sodium chloride (Nacl) & zinc sulphide (Zns) crystal. The emergent ~~the~~ rays are made to fall on a photographic plate P (screen). The diffraction pattern consists of a central spot at O and a series of spots arranged in a definite pattern about O.

This symmetrical pattern of spots is known as Laue pattern and proves that X-rays are ~~at~~ electro-magnetic waves. According to Bragg, the spots are produced due to the ~~at~~ reflections of some of incident X-rays from the various sets of parallel crystal planes which contain large no of atoms. The Laue Spot expt has established following two important facts

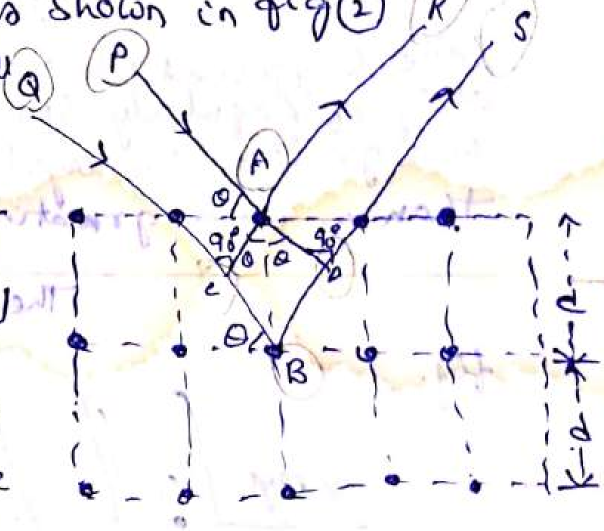
① x-rays are electromagnetic waves of extremely short wavelengths.

② The atoms in a crystal are arranged in a regular 3D lattice. He was awarded Nobel prize in 1914.

→ Bragg's Law :-

Let us consider a set of parallel lattice planes of a crystal separated by a distance d apart, suppose a narrow beam of x-rays of wavelength λ be incident upon these planes at angle θ as shown in fig ②

The beam will be reflected in all directions by the atoms of various atomic planes. Because the refractive index of the matter of the crystal is very nearly equal to unit. The path difference b/w two reflected waves must be an integral multiple of wave length.



Consider a ray PA reflected at atom A in the direction AR from plane 1 and another ray QB reflected at another atom B in the direction BS. Now from the atom A draw two ⊥er AC and AD on the incident ray QB and reflected ray BS resp. The path difference b/w these two rays is (CB+BD). The two reflected rays will be in phase & out of phase will depend upon this path difference.

When the path difference (CB+BD) is a whole wavelength λ & multiple of whole wavelength $n\lambda$ then the two rays will reinforce each other and produce an intense spot. Thus, the condition of reinforcement is

$$CB + BD = n\lambda$$

from fig $CB = BD = d \sin\theta$.

$$\boxed{2d \sin\theta = n\lambda} \longrightarrow (1)$$

where $n = 1, 2, 3 \ldots$ for first order, second order, third order
This relation is known as Bragg's Law.

for maxima $\sin\theta_1 = \lambda/2d$

second " $\sin\theta_2 = 2\lambda/2d$

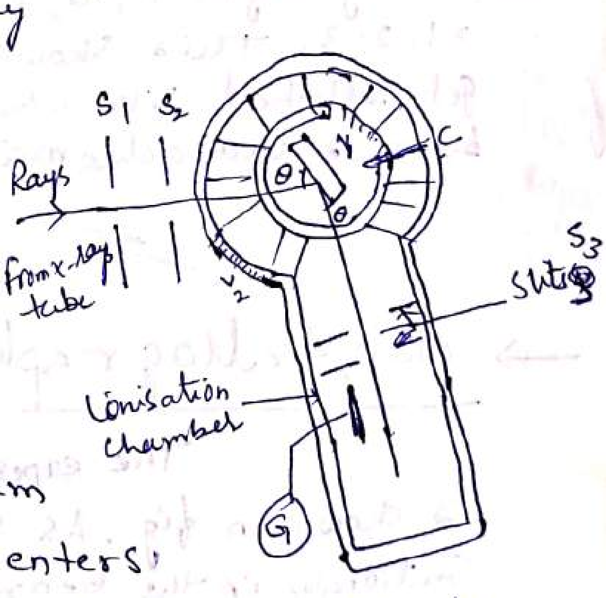
" " $\sin\theta_3 = 3\lambda/2d$

12

Theory:- When a beam of monochromatic X-rays falls on a crystal each atom becomes a source of scattering radiations, At certain glancing angles reflections from these sets of parallel planes are in phase with each other and hence they reinforce each other to produce maximum intensity. For other angles the reflections from diffrent planes are out of phase and hence they reinforce to produce either zero or extremely feeble intensity.

→ Bragg's ~~X-ray Spectrometer~~ Law experimental technique:-

The schematic arrangement of Bragg's Spectrometer is shown in fig. X-rays from X-ray tube are narrowed to obtain a fine pencil of beam by passing them through slits S_1 and S_2 provided in lead screens. The beam is now allowed to fall on a crystal C mounted on a circular turn table of the spectrometer.

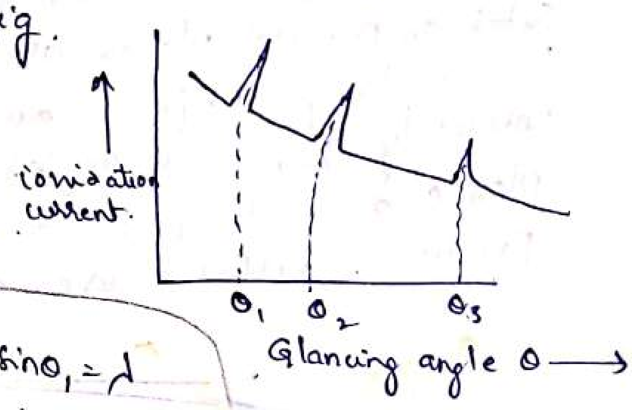


The reflected beam then passes through slits S_3 and enters in ionisation chamber. Ionisation chamber is simply a container for gas or vapour with two electrodes. The ionisation chamber is mounted on a special movable arm about the same axis of crystal.

when the turn table rotates through an angle θ, the ionisation chamber turns through 2θ. In this way the beam is always reflected into the ionisation chamber whatever may be the glancing angle at the surface of the crystal.

The x-rays entering the ionisation chamber ionise the gas which causes a current to flow b/w two electrodes which can be measured by galvanometer G. The ionisation current is measured for different values of glancing angle θ. A plot is then obtained b/w θ and the ionisation current. The graph is shown in fig.

For certain values of glancing angle θ, the intensity of ionisation current increases. From Bragg's equation that



B

$2d \sin\theta = n\lambda$

for 1st order $n = 1$ and $2d\sin\theta_1 = \lambda$

" $n = 2$ and $2d\sin\theta_2 = 2\lambda$

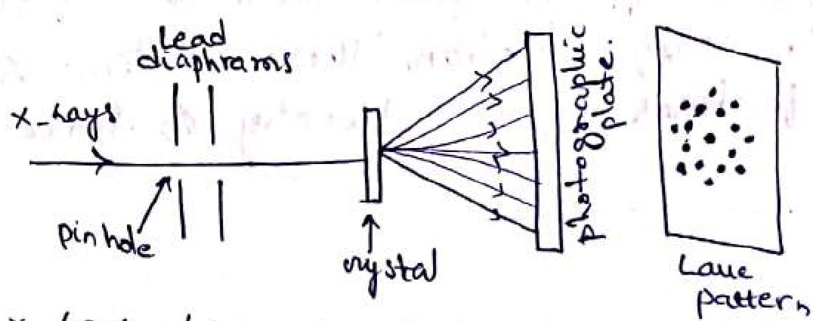
" $n = 3$ " $2d\sin\theta_3 = 3\lambda$

$\therefore \sin\theta_1 : \sin\theta_2 : \sin\theta_3 = 1 : 2 : 3.$

from the graph glancing angle θ and ionisation curve the glancing angles θ_1, θ_2, θ_3. It can be seen that $\sin\theta_1 : \sin\theta_2 : \sin\theta_3 = 1 : 2 : 3$. This shows that the assumption that x-rays get reflected like ordinary light is justified. Thus x-rays beam is monochromatic.

→ crystallography By Laue Method:-

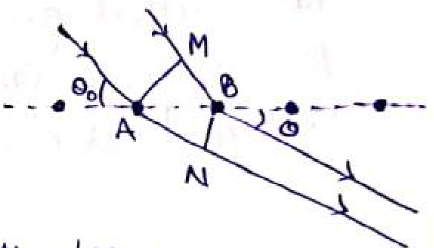
The experimental arrangement of Laue method is shown in fig. As shown in fig. the crystal is held stationary in the beam of x-rays. After passing through the crystal the x-rays are diffracted and recorded on photographic plate. The x-rays before passing through the crystal are limitted to a fine pencil by a slit System, Here the diameter of the pinhole is important because smaller is the diameter, sharper is the interference.



The x-rays which penetrate the crystal are scattered from different atomic diffraction centres. This is possible because there is a whole range of wavelengths in continuous spectrum and there will be discrete values of λ which satisfy the Bragg condition.

The diffraction pattern consists of a central spot and a set of spots arranged in a definite pattern about central spot. The symmetrical pattern of spot is known as Laue pattern.

Suppose monochromatic x-rays of wavelength λ are incident on the space lattice. The electrons in the atoms at the lattice points scatter x-rays in all directions coherently. Let θ_0 be the complementary angle of incidence and θ the complementary angle of diffraction as shown in fig ② The path difference b/w the two diffracted rays $(AN - BM)$ and the diffracted rays will have maximum intensity if



$$AN - BM = n_1\lambda \longrightarrow (1)$$

n_1 = integer.

from fig $AN = AB\cos\theta = a\cos\theta$

$BM = AB\cos\theta_0 = a\cos\theta_0$

$$a(\cos\theta - \cos\theta_0) = n_1\lambda \longrightarrow (2)$$

where $\cos\theta_0$ and $\cos\theta$ by α_0 and α, known as directional cosines of the incident and diffracted rays. Hence

$$a(\alpha - \alpha_0) = n_1\lambda \longrightarrow (3)$$

and $a(\beta - \beta_0) = n_2\lambda$, $a(\gamma - \gamma_0) = n_3\lambda \longrightarrow (4)$

where $n_1 n_2 n_3$ are integers. These are called Laue eq's. This means that every diffracted ray is characterised by 3 integral no's n_1 n_2 n_3 are called the order number. These are different from the orders of the spectra which occur in Bragg's relation. These relations represent the essential features of the theory of Space Lattice. the slits of

---

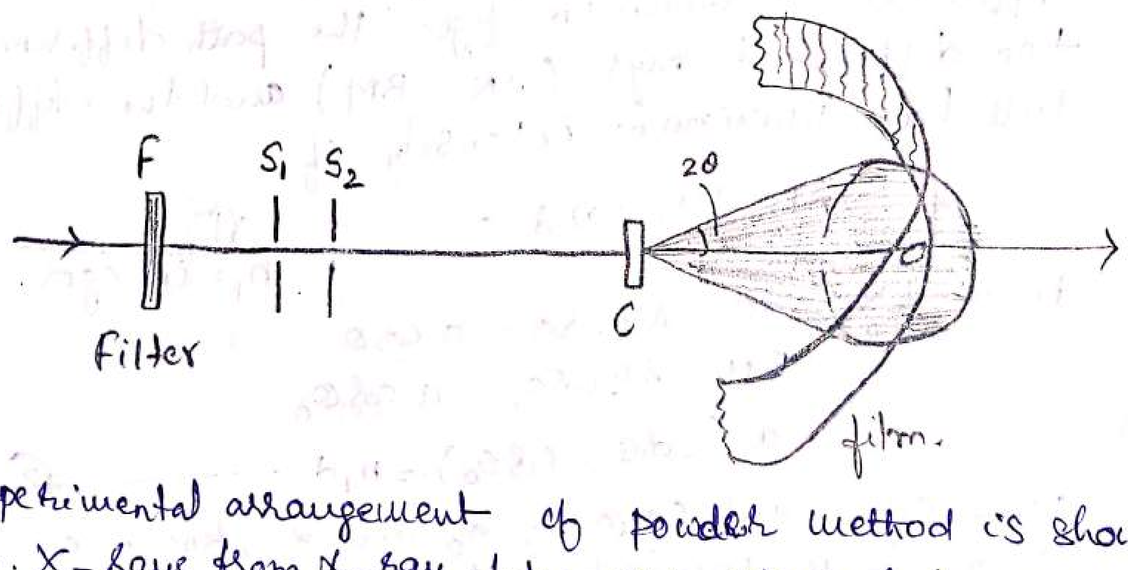
Diffraction :- When light passes past an opaque obstacle, some of it bends round to obstacle into the shadow region and the intensity of light is reduced in some parts of the non-shadow region.

grating :-

(15)

→ Powder diffraction Method :-

powder crystal method is a Standard technique to study the structure of micro crystals. This method gives information regarding the size of crystals, presence of impurities, distortion, preferred orientation of crystal etc.



The experimental arrangement of powder method is shown in fig. X-rays from X-ray tube are allowed to pass through a filter F which absorbs all wavelengths expect one. In this way a monochromatic beam of x rays is obtained.

The beam is collimated by passing it through two fine slits S_1 and S_2 cut in two lead plates. This fine pencil of X-rays is made to fall on the powdered specimen C. The specimen is located at the centre of drum shaped casette with photographic film at the inner circumference.

The basic principle underlying this powdered technique is that in the powder, million of micro crystals have all possible random orientation. Among these very large no. of micro crystals there will always exist some crystal whose lattice planes are so oriented to satisfy the Bragg's relation $n\lambda = 2d\sin\theta$

As the parallel lattice planes with a given spacing d and same value of n and θ occur in all positions around the axis of the incident beam, the reflected rays produces a cone with semi-vertical angle 2θ. For various set of d and n, various cones of rays are obtained; one such cone is shown in fig (a).

The scattered X-rays are incident on the photographic film and form a series of concentric circular rings. Radii of these rings can be ~~calculated~~ used to find glacing angle. ~~Now~~ The pattern recorded on the photographic film is shown in fig (b). When the film is laid flat. Due to the narrow width of the film, only parts of circular rings are register on it. The curvature of arcs reverses when the angle of diffraction exceeds 90°.

Let ~~of~~ l_1, l_2, l_3 etc be the distances b/w

16

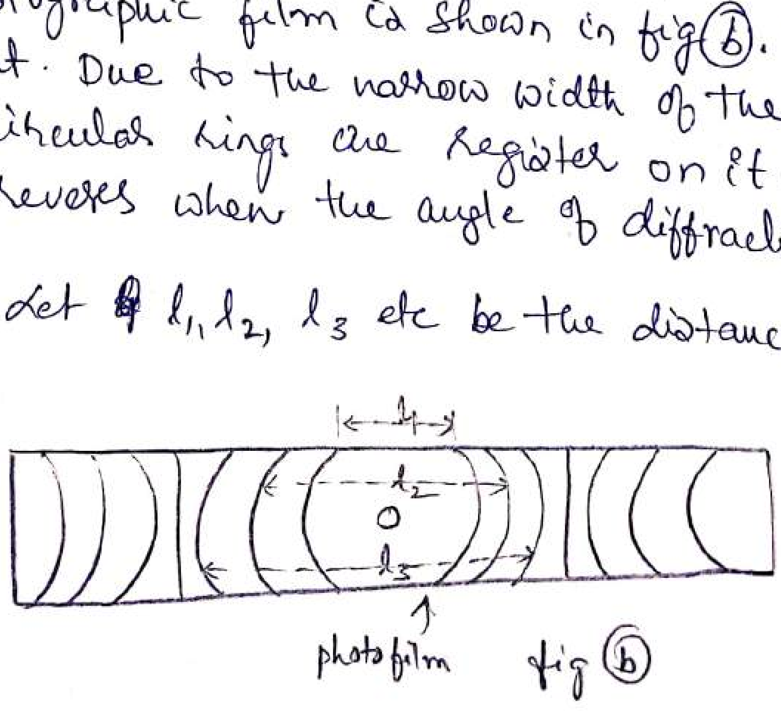


fig (b)

Let l_1 l_2 l_3 etc be the distance b/w symmetrical lines on stretched photographs and D, the diameter of cylindrical film then

$$\frac{l_1}{\pi D} = \frac{4\theta_1}{\cancel{360^\circ}_{90^\circ}} \quad \& \quad \theta_1 = \frac{90^\circ}{\pi D} l_1$$

||ly $\theta_2 = \frac{90^\circ}{\pi D} l_2$ and $\theta_3 = \frac{90^\circ}{\pi D} l_3$.

Using these values of θ in Braggs formula, interplanar spacing d can be calulated.

17

$$\frac{l_1}{\pi D} = \frac{\cancel{4}\theta_1}{\cancel{360}_{90}}$$

$90 . l_1$

V. 7 - Super Conductivity ①

The phenomenon of Super conductivity was discoved by Kamerlingh Onnes in 1911. He obsewed that electrical resistivity of pure mercury is 4.2 K and it is new state which is called super conductivity state.

The temp at which the resistance disappers is called the transition temp & critical temp.

When a substance losses its electrical resistance i-e a current can continue through it without altering its value, the phenomenon is known as Super-Conductivity.

When the electrical resistance of a ~~supstan~~ Substance drops suddenly to zero when its specimen is cooled below a certain temp, the phenomenon is known as superconductivity.

18

→ Experimatal Facts

Following are the few basic experimental facts of superconducting materials.

① At ~~to~~ room temp Super conducting material have greater resistivity.

② Transition temp T_c is decreases with increasing atomic weight of the isotopes.

③ The super conducting property does not change by adding impurities

④ There is no change in crystal structure as revealed by X-rays diffraction

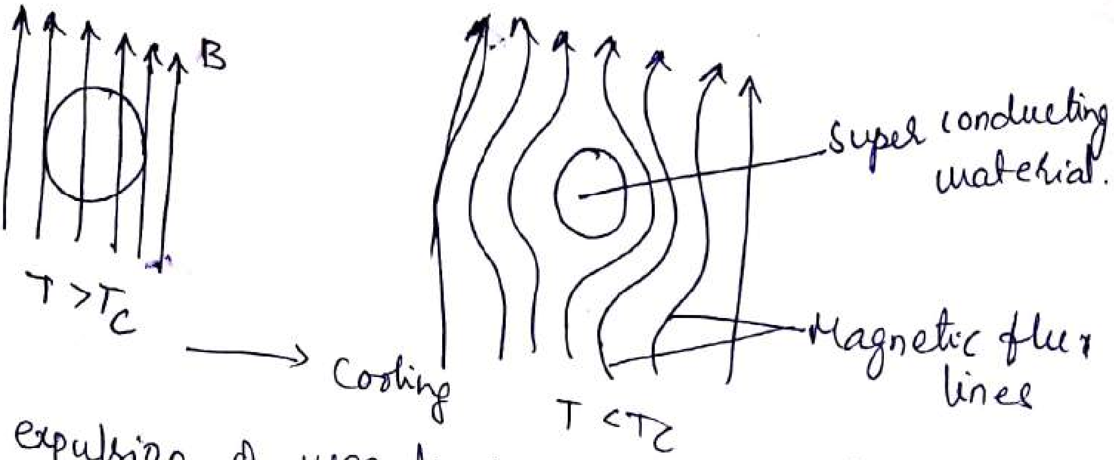
⑤ ~~x~~ The Super Conductor is characterized by zero electrical resistance

⑥ The entropy of all super conductors increases as one passes from superconducting state to normal state.

Meissner effect :- (flux Exclusion)

In 1933 Meissner observed at if a Super Conductor is cooled in a magnetic field below critical temp corresponding to that field then the lines of induction are expelled from the material. This effect is called as Meissner effect.

19



The expulsion of magnetic lines of force from a super-conducting material when it is cooled ~~below that~~ below the transition temp in a magnetic field is called Meissner effect.

1. Meissner effect is Reversible when the temp is increased below T_C the flux penetrates through the Specimen and the Substance comes to its normal state.
2. A Super conductor is a perfect diamagnetic. The reason is that the magnetic induction B in a super conductor is zero. For Superconductors,

$$B = \mu_0 (H + M) = 0 \quad \& \quad M = -H \longrightarrow (1)$$

H - Magnetising field Intensity
M - Intensity of magnetization
The magnetic susceptibility χ is given by

$$\boxed{\chi = \frac{M}{H}} \longrightarrow (2)$$

from eq (1) and (2) we get.

$$\chi = \frac{-H}{H} = -1 \longrightarrow (3)$$

This is the max value for the Susceptibility of a dia magnetic material. Then a superconductor is a perfect diamagnetic

Maxwell's equations is given by

$$\nabla \times E = -\frac{\partial B}{\partial t} \longrightarrow (4)$$

20

According to ohms Law

$$V = IR \quad \text{and} \quad E = V/d.$$

$$E = \frac{IR}{d} = \frac{(JA)R}{d} = J\left(\frac{AR}{d}\right)$$

$$\cancel{E = \frac{IR}{d} = \frac{(JA)R}{d}\ \rho = J\rho}$$

$$E = J\rho \quad \text{where} \quad \rho = \left(\frac{AR}{d}\right)$$

For finite J and zero ρ, E should be zero

from eq (4) $\frac{\partial B}{\partial t} = 0$ or B = const.

The Superconductors should be judged by both the conditions independently. So for a Super conductor 1) has zero resistance below critical temp 2) shows Meissner effect below critical temp.

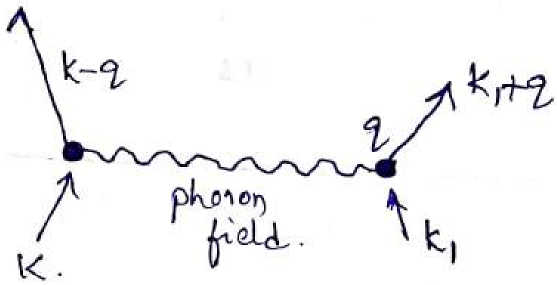
The difference b/w a perfect conductor and a Super conductor is that the former is only an ideal conductor, while the later is simultaneously an ideal conductor and an ideal diamagnet.

→ BCS Theory :-

In 1952 Bardeen, cooper, and Schrieffer gave a theory to explain the phenomena of superconductivity which is the known as BCS theory. The theory is based on the formation of cooper pairs.

When a current flows through a super conductor and an electron (−ve charge) comes near to the +ve ion core of the lattice, then the electron experiences an attractive force. Due to interaction b/w electron and ion core, the ion core is slightly displaced. This is known as lattice distortion. The distortion in the lattice then travels away as a mechanical wave (phonons).

Hence there is a force of attraction b/w second electron and phonon. In this way two electrons interact with each other through the lattice vibration and this process is called as electron- lattice - electron in integration via a phonon field. This is shown in fig.



Due to interaction we have two electrons with wave vector K−q and K,+q. The pair of electrons is called a cooper pair.

Cooper pair is a bound pair of electrons formed by the interaction b/w the electrons in a phonon field. The two electrons which pair up have opposite momenta and spin.

When the pair of electrons flow in the form of copper pair they do not encounter any scattering and resistance factor vanishes. Conductivity becomes

infinity

infinity which is named as Super conductivity

→ Applications of super conductivity :- (expand)

1. power transmission
2. Super Conductivity magnets
3. Electrical applications (cryotron)
4. Maglevlic vechicles
5. very strong magnetic fields
6. SQUIDS (Super conducting quantum interfaberence device)
7. For progress of computer technology.

(Explanation need)

22

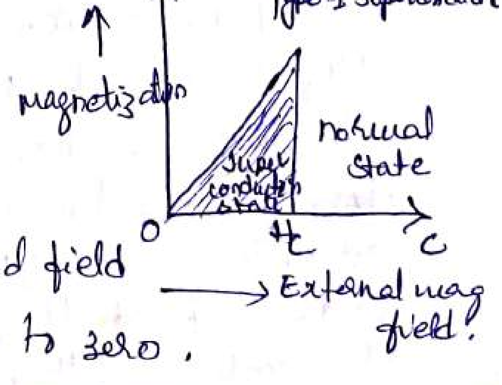
→ Type-I, Type-II Supercondutors :-

Based on magnetic behaviour the Super conductors are classified into following two categories.

1. Type-I Super Conductor & soft super conductor
2. Type-II Super condutor & hard super conductor.

Type-I Super conductor :-

The dependence of magnetization of a Super condultor of Type-I as a function of external field H as shown in fig. from fig upto the critical field strength (H_c) the magnetization of Super conductor grows in proportion to the external field.



As soon as the applied field H exceeds H_c the magnetization drops to zero.

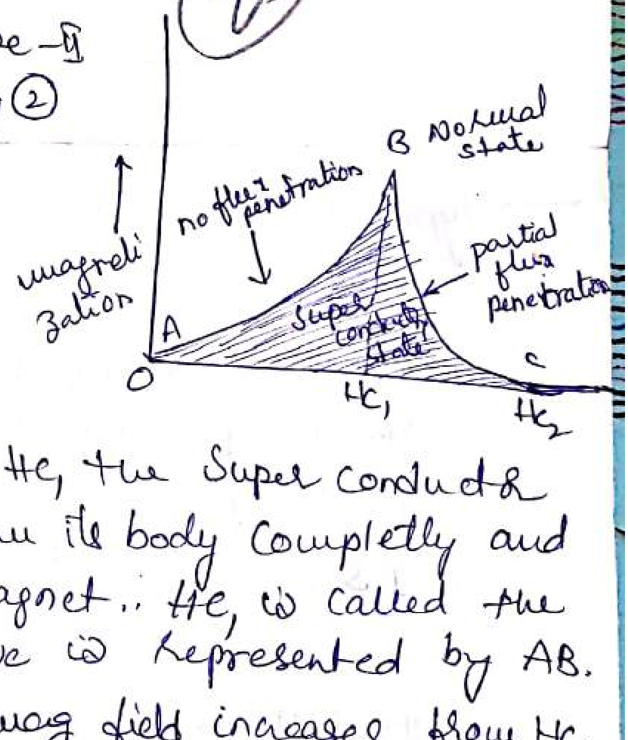
So type-I super conductor is one in which the transition from super conducting state to normal state in presence of magnetic field occurs sharply at the critical value. H_C.

In the presence of external mag field $H < H_c$, type I superconductor in super conducting state is a perfect diamagnet, when H exceeds H_c the super conductor enters the normal state i-e it loses its diamagnetic property. The critical field value H_c for type-I super conductor is found to be very low. Aluminium, lead and indium are examples of Type-I.

23

Type-II Super Conductors :-

The magnetization curve of type-II super conductor is shown in fig ②. The type-II super conductor is characterized by two critical magnetic fields Hc_1 and Hc_2. The description of the curve is as follows.



① For the field strength below Hc_1 the super conductor exples the magnetic field from its body completly and behaves as a perfect diamagnet. Hc_1 is called the lower critical field. The curve is represented by AB.

As the mag field increases from Hc_1, the mag lines begin to penetrate the material. The penetration increases until Hc_2 is reaches. Hc_2 is called upper critical field. At Hc_2 the magnetization vanishes completly. After Hc_2 the material ~~returns~~ turns to normal state. The most important advantage of Type II superconductor is the value of critical field Hc_2 which is many more times higher than the value of Hc_1 for type I superconductor.

→ Isotope effect :-

The first observation was first made by Maxwell's and others, who used mercury isotopes. The isotopic mass enter in the process of the formation of the superconducting phase of the electron states only through the electron - phonon interaction.

By BCS theory $T_c \propto M^{-\beta}$. & $T_c \approx M^{-1/2}$

$\beta = +0.5$ was through to be valid for most materials.

This is called the isotope effect. The critical temp is proportional to frequency of oscillation of the ions around their equilibrium positions

$T_c \sim \omega$

24

Scanned with CamScanner

Major II.

Introduction to Nuclear and particle physics.

UNIT- III

Nuclear Reactions and Nuclear Detectors.

→ **Nuclear Reactions - Types of reactions :-**

The following are the simple types of Nuclear reactions

1) **Elastic Scattering:-**

The incident particle strikes the target nucleus and leaves without energy loss. Eg:- Scattering of α-particles in gold

$${}_{2}He^{4} + {}_{79}Au^{197} \longrightarrow {}_{79}Au^{197} + {}_{2}He^{4}$$

2) **Inelastic Scattering:-**

The scatterd particle may loss K.E in an elastic collision with nucleus. This tends to increase in the internal energy of the nucleus.

$${}_{3}Li^{7} + {}_{1}H^{1} \longrightarrow ({}_{3}Li^{7})^{*} + {}_{1}H^{1}$$

* Start indicates after scattering, nucleus is left in an excited state.

3) **Disintegration :-**

On striking the target nucleus the incident particle is absorbed and a different particle is ejected. The product nucleus differs from target nucleus.

$${}_{7}N^{14} + {}_{2}He^{4} \longrightarrow {}_{8}O^{17} + {}_{1}H^{1}$$

(2) (8) way

4) photo disintegration:-

The γ-rays are absorbed by the target nucleus, exciting it to a higher quantum state.

$$_1H^2 + \gamma \longrightarrow {}_1H^1 + {}_0n^1.$$

5) Radiative capture:-

A particle may combine with a nucleus to produce a new nucleus which is in an excited state. The excess energy is emitted in the form of γ-ray photons. This type of process is known as radiative capture.

$$_{12}Mg^{26} + {}_1H^1 \longrightarrow ({}_{13}Al^{27})^* \longrightarrow {}_{13}Al^{27} + \gamma.$$

6) Direct Reactions:-

A collision of an incident particle with the nucleus may immediately pull one of the nucleons out of the target nucleus by the so called pickup reaction.

In the inverse process a bombarding particle composed of more than one nucleon may loss one of them to the target by the so-called stripping reaction.

$$_{29}Cu^{63} + {}_1d^2 \longrightarrow {}_{29}Cu^{64} + {}_1H^1$$

7) Heavy Ion Reactions:-

Nuclear reactions induced by heavy ions exhibit the characteristics both of compound nucleus and of the stripping and pickup mechanisms, Eg.

$$N^{14} + Pb^{207} \longrightarrow N^{13} + Pb^{208}.$$

(8) Spontaneous decay:- β and α-decay processes may be of this type of nuclear reactions. In these reactions the total energy of the system is not under the experimenter's control. (8)

(9) Spallation Reactions :-

on capture a incident particle a heavy nucleus has sufficient energy for the ejection of several particles. Such a reaction is known as spallation reactions.

Eg:- Nuclear fission.

$$ {}_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{40}Zr^{98} + {}_{52}Te^{136} + 2\,{}_{0}n^{1} $$

(10) High energy Reactions:-

In the energy range about 150MeV, spallation process merges into new kinds of reactions in which new kinds of particles (mesons, strange particles) are produced along with neutrons and protons.

→ Conservation Laws:-

In any nuclear reactions, certain quantities must be conserved.

1) Conservation of Energy:-

The total energy of the products including both mass energy and K.E of the particles plus the energy involved must be equal to the mass energy of the internal ingredients plus the K.E of bombarding particles.

(4) 2) Conservation of Momentum:-

The total linear momentum of the products must be equal to the linear momentum of the bombarding particle.

3) Conservation of Angular momentum:-

The total angular momentum l comprising the vector sum of the intrinsic angular moment S and relative orbital momentum l of the products must be equal to the total angular momentum of the initial particles.

4) Conservation of charge:-

The total ~~ch~~ electric charge of the products must be equal to the total electric charge of initial particles.

5) Conservation of Nucleons:-

The law of conservation of nucleons states that the ~~new~~ nucleons can neither be created nor can be destroyed so that no of nucleons minus the no of anti nucleons in the universe remains constant.

6) Conservation of Spin:-

The Spin character of a closed system cannot change i.e the statistics remains same as that existed before reactions.

7) Conservation of parity:-

The parity of the system determined by the target

Conse

nucleons and bombarding particle must be conserved throughout the reaction.

The total parity of the system is the product of intrinsic parities of the target nucleus and bombarding nucleus.

No violation of parity has been observed in strong nuclear forces and does not appear in weak interactions.

8) Conservation of Isotopic Spin:-

The invariance of the nuclear Hamiltonian function towards the charge, character of the nucleons can be expressed analytically as an invariance towards rotational shifts of the axes in isotopic spin space and there should correspondingly exist a conservation law for the isotopic spin of a nuclear system.

→ Reaction Energetic & Threshold energy:-

Nuclear reactions are divided energetically as

1) Exoergic Reactions 2) Endoergic Reactions.

1) Exoergic Reactions:- These reactions are possible even for $E_x = 0$. Thus for $E_x \to 0$ in equation (1) gives

$$\sqrt{E_y} = u \pm \sqrt{u^2 + v} \quad \text{——} (1)$$

Where $u = \dfrac{\sqrt{m_x m_y E_x}}{M_y + m_y} \cos\theta$, and $v = \dfrac{M_y Q + E_x(M_y - m_x)}{M_y + m_y}$.

⑥ $$E_y = \left(\frac{Q M_y}{m_y + M_y}\right) \quad (Q>0) \longrightarrow ②$$

The kinetic energy E_y is ~~max~~ same for all angles θ. Because total momentum is effectively zero, when $E_x \to 0$.. In this case

$$Q = E_Y + E_y \text{ and } \theta + \phi = 180^\circ$$

In this commonly projectile x is lighter than the product nucleus Y. Thus v is +ve for all values of the bombarding energy and E_y depends on $\cos\theta$ and is smallest in the backward direction $\theta = 180^\circ$.

2) <u>Endoergic Reactions</u> :- All endoergic reactions have negative Q-values. When $E_x \to 0$ in eq ① gives $u^2 + v = -ve$ and hence $\sqrt{E_y}$ is imaginary.. It means that these reactions are not possible. The smallest value of E_x at which reactions can take place is called the <u>threshold energy</u>.

These reaction first becomes possible when E_x is large enough to make $u^2 + v = 0$.

$$E_x = -Q\left[\frac{m_y + M_y}{m_y + M_y - m_x - (m_x m_y / M_y)\sin^2\theta}\right]$$

At $\theta = 0$, E_x is min and is the threshold energy.

$$\therefore (E_x)_{th} = -Q\left[(M_y + m_y)/(M_y + m_y - m_x)\right]$$

Nuclear Cross-section :-

The propability of occurance of a particular nuclear reaction is described by the effective cross-section for that process.
The cross section may be defined as

1) The probability that an event may occur when a single nucleus is exposed to a beam of particles of total flux one particle per unit area.

&

2) The probability that an event may occur when a single particle is shot free at a target consisting of one particle per unit area.

The unit for cross section is barn ($= 10^{-28} m^2$). It falls b/w 10^{-27} and $10^{-28} m^2$.
There are two types of cross sections.

1) partial cross-section
2) Differential cross section.

partial cross-section :-

The total nuclear cross section is the effective area possesed by a nucleaus for removing the incident particles, from a collimated beam by all possible processes. This can be written as

$$\sigma_t = \sigma_s + \sigma_r.$$

where σ_t - total cross section
σ_s - cross section that produces measurable scattering
σ_r - the reaction cross section.

(8) Scattering cross section can be classified as a) inelastic scattering and
b) elastic scattering processes.
Thus

$$\sigma_s = \sigma_{el} + \sigma_{inel}$$

and

$$\sigma_{inel} = \sigma_1 + \sigma_2 + \sigma_3 + \cdots$$

IIIly

$$\sigma_{el} = \sigma' + \sigma'' + \sigma''' + \cdots$$

2) <u>Differential Cross-Section:-</u>

The cross section which defines a distribution of emitted particles w.r.t the solid angle is called differential cross section. It is defined as $\frac{d\sigma}{d\Omega}$ and the partial cross section for a given process is given by

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$$

Hence the nuclear cross section can be defined as the no of light product particles per unit time per unit incident flux and per target Nucleus.

$$\frac{N}{I} = \frac{\sigma n A \Delta x}{A} \quad \& \quad \sigma = \frac{N}{\left(\frac{I}{A}\right)\left(n A \Delta x\right)}$$

Nuclear detectors :- Geiger Muller counter
Scintillation counter
Cloud chamber

The instruments which are used for the detection of nuclear radiations are

1. Ionization chamber
2. proportional counter.
3. Giger-Muller counter
4. ~~Col~~ Cloud chamber
5. Bubble chamber
6. Scintillation counter
7. Semiconductor counter
8. photographic emulsion
9. Spark chamber:

Solid State detector

1. The Ionization chamber depends upon the principle that charged particles in motion produce ionization in a gas.

2. The proportional counter consists of a cylindrical metal chamber with a thin wire situated along its axis and insulated from it. When a charged particle enters in metallic chamber, ionization of gas takes place resulting in an ion pair formation.

5. Bubble chamber is based on the property of super heated liquid. When the pressure is suddenly reduced to atmospheric pressure, the ~~ioniz~~ ionising particle passes the superheated liquid, it leaves a trails of ions behind it. The track of bubbles can readily illuminated and photographed.

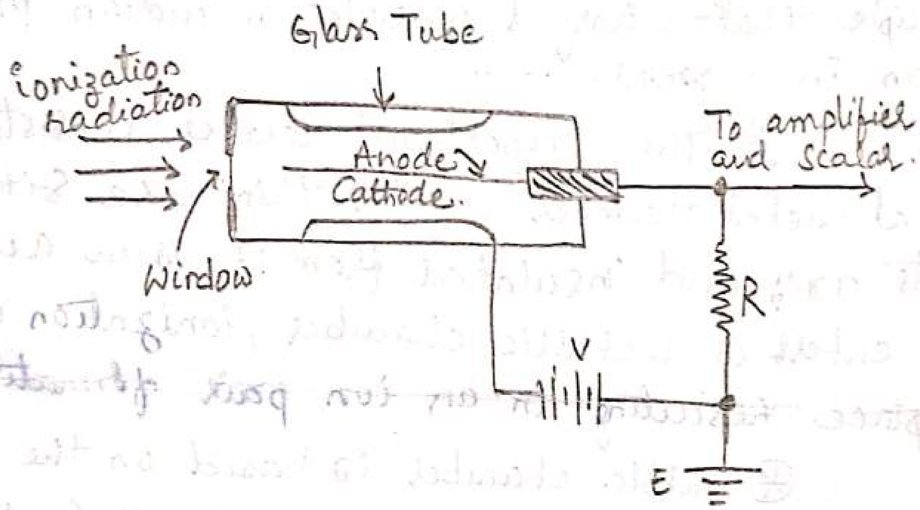
6. Scintillation counter:- When high energy charged particles pass through active fluorescent material, some of its atoms are excited, and return to ground state photons are emitted. The Scientillation counter can count 10^6 particles per second.

8. photographic emulsion:- powell in 1933 developed nuclear emulsion plates which are very sensitive to all types of charged particles. When an ionising particle travels through the sensitive emulsion of photographic plate is developed to see the tracks

→ Geiger–Muller Counter:-

The Geiger-Muller counter consists of fine wire placed (tungsten) along the axis of a hollow metal cylinder electrode enclosed in a thin glass tube.

The tube contains a mixture of 90% argon at 10cm pressure and 10% ethyl alcohol vapour at 1cm pressure. At one end of the tube, a window covered with thin mica sheet as shown in fig where the ionizing particles & radiation enter into the tube.

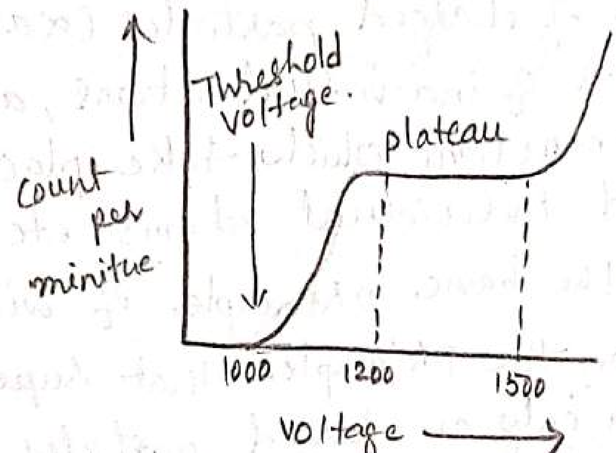


A d.c potential of about 1200 volts is applied between cathode and the wire which acts as an anode. A high resistance R is connected in series with battery. When a charged particle passes through the counter, it ionizes the gas molecules. This causes an ionization current which depends upon the applied voltage. Thus the larger no of secondary electrons are produced.

The incoming particle serves the purpose of triggering the release of an avalanche of secondary electrons. The electrons quickly reach the anode and causes ionization current. In this way a brief pulse of current flows through resistance R. This current

Geiger - Muller Counter:- (7)

creates a potential difference across R. As each incoming particle produces a pulse, hence the no of incoming particles can be counted.



The successful operation of G.M counter depends upon the proper voltage to electrodes. fig (2) represents the counts per minute as a function of voltage.

As applied voltage is increased to ~~about 1200 volts, the no of impulses remains constant~~ over a certain region known as plateau. If the voltage is increased above this region, a continuous discharge will take place.

The G.M Counter can count about 500 particles per second. The counting rate depends upon 1) dead time 2) recovery time 3) paralsis time.

The slowly moving +ve ions take about 100 micro seconds to reach the cathode. This time is known as dead time. After dead time the tube takes nearly 100 micro seconds before it regains its original working conditions. This time interval is known as recovery time. The sum of dead time and recovery time is known as paralysis time which of course is 200 micro seconds.

The GM counter is very useful for counting β-particles, γ-ray intensities and not for α-particle counting because of their low energy as window can not be made thin enough to pass them.

wilson window

→ Wilson Cloud chamber:-

In 1911 CTR wilson devised an instrum known as cloud chamber by which it is possible to detect and record the paths of charged particles (α and β rays) Studying the behaviour of individual atoms, analysing the complicated interactions which take place between charged particles and individual atoms etc.

The basic principle of wilson cloud chamber is based on the principle that supercooled vapour condensed ~~at~~ only on charged particles and if the charged particles are not present in vapour phase. i.e they remain they do not condense.

In wilson cloud chamber, ions act as nuclei for condensation of super-saturated water vapours. The ions are produced by the passage of high energy particles like α and β particles etc, through the chamber. The particles produce ionisation of the air & gas contained in the chamber.

There are two types of wilson cloud chambers. ① Expansion type wilson cloud chamber and ② diffusion type wilson cloud chamber.

fig① shows the expansion type wilson cloud chamber. It consists of an air-light cylinder C provided with a movable piston P and upper end covered with a glass plate G. The chamber contains a mixture of ~~alcho~~ alcohol vapour and air.

The chamber is illuminated by mercury vapour lamp L whose light enters in chamber by a window. α & β rays emitted by a radioactive substance which enters into the chamber by a side

Wilson cloud chamber

window as shown in fig ① A photographic camera is adjusted on the upper side of the chamber.

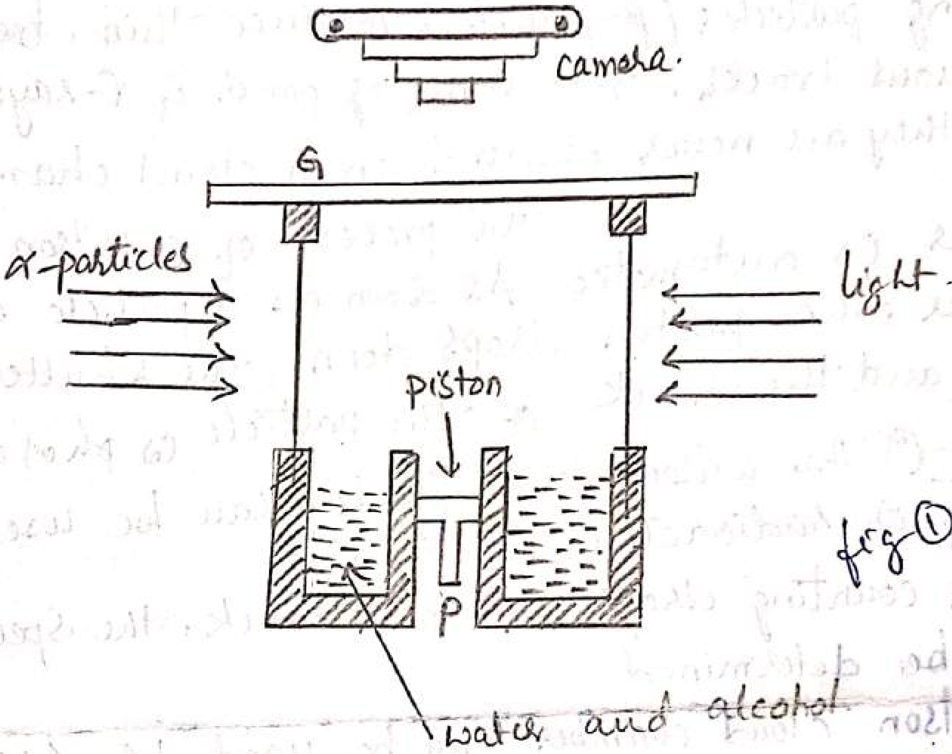


fig ①

Working:- The volume of the chamber is suddenly increased by pulling back the piston. The adiabatic expansion of the gases takes place and air cooling is produced. The air in the chamber now becomes super saturated with the result that alcohol vapour condenses into drops which form ~~fog~~ in fog the chamber.

When α & β rays are allowed to pass in the chamber, they ionise the gas. Now −ve and +ve ions are formed all along the path of these rays.

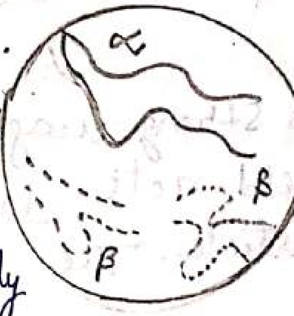


fig ②

If at the same time expansion takes place, fog drops will be formed on newly created ions. The drops are clearly visible when the chamber is illuminated by light. The tracks can be photographed with the help of camera. Different particles produce different types of ~~tak~~ tracks as shown in fig (a) Heavy, slow and highly-ionizing pa (α-particles) produce short, broad, densely packed straight line tracks.

On the other hand, the light, slow and less ionizing particles (β-particles) produce thin, beaded and tortuous tracks. But ionizing power of γ-rays is very low and they are never observed in a cloud chamber.

The process of a wilson cloud chamber is automatic. As soon as a particle enters the chamber, the piston drops down, the shutter of camera opens and the track of the particle is photographed.

Uses:- (1) The wilson cloud chamber can be used for the study of radioactive radiations

(2) By counting drops in cloud track, the specific ionisation can be determined.

(3) Wilson cloud chamber can be used for recording the tracks of ionising particles.

(4) By seeing the direction of curvature of track in the magnetic field, sign of electric charge of ionising particle can be determined.

(5) Wilson cloud chamber can be used for the determination of energy of various particles

The ~~pricipl~~ principle for this determination is described below.

The wilson cloud chamber is placed in strong magnetic field in such a way that magnetic field acts ⊥er to the direction of motion of the particle. The particle experiences a force F given by

$$F = qVB$$

where q = charge on the particle

V = velocity of the particle

B = mag field Strength

Due to this force, the particle describes a circular path. Now the particle attains a centripetal force mv^2/r. Thus

$$\frac{mv^2}{r} = qvB$$

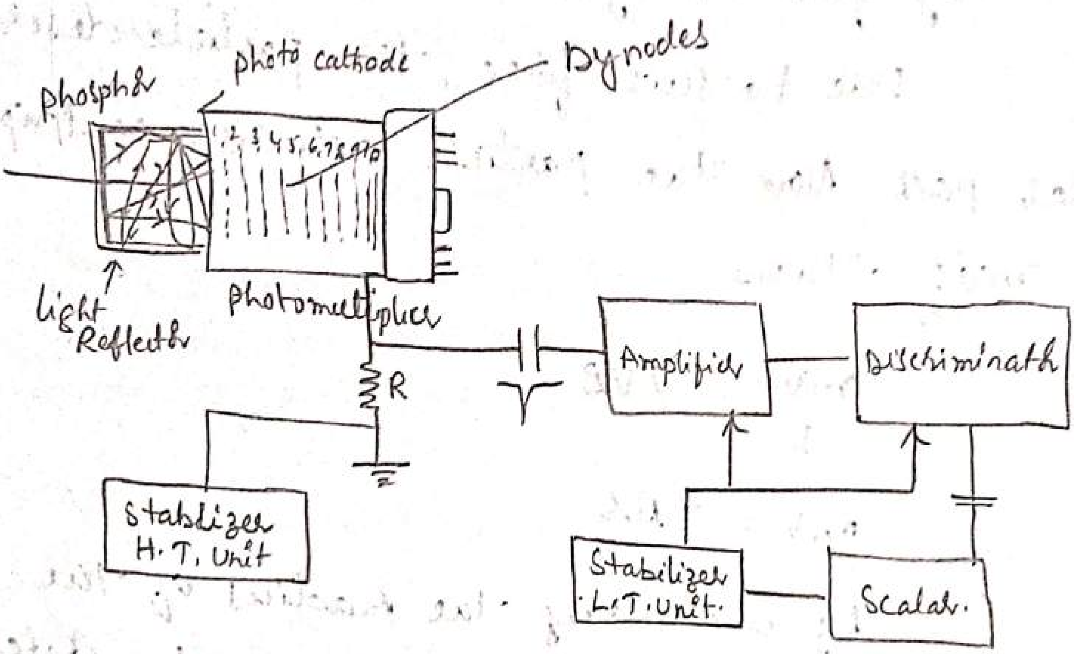
$$mv = qBr.$$

By measuring the radius of the path, the momentum of the particle can be determined. From Momentum, the energy can be calculated.

→ Scintillation counter :-

To count α-particles by observing flashes of light (Scintillations) generated in diamond by them, was made by E. Regener in Germany in 1908.

A Schematic diagram of a ~~Scin~~ Scintillation counter, as shown in fig, consists of luminescent material reflecting layer such as aluminium foil enclosing the luminescent substance to facilitate the collection of light light pipes, the photomultiplier tube, amplifier, voltage discriminator and an electrical circuit to record the output pulses.



The following are the components of ~~Scint~~ Scintillation counter 1) photomultiplier tube, Scintillator

In Scintillation counters human eye is replaced by an electronic device called a photomultiplier tube. It is highly sensitive photocell, converting light energy into electrical energy. A semi transparent Cs - Sb cathode is deposited on the inside of the end of a high vacuum envelope.

The electrons ejected from this cathode are accelerated towards the electrode D_1 and are collected by the electrode. These are focused on an electron D_2. The process is repeated in several stages by focusing the electrons from one surface to the next. These collecting electrodes D_1 D_2 -- etc are called dynodes. (13-16 dynodes in photo multiplier tube)

The dynodes has a venetian a ~~by~~ layer of material with a high secondary emission coefficient such as $Cs-Sb$ & $Ag-Mg$.

The Anode A is connected to the +ve voltage supply through a series resistance R. Because of voltage drop a negative o/p pulse is produced by flash of light falling on the photo cathode. This pulse can be recorded by scalars.

At room temp there will be an appreciable emission of thermionic electrons from the cathode to produce undesirable current known as dark current.

If τ be the mean life of the scintillator the no of light photons in time t after the ionization radiation has arrived is given by

$$n = \text{const} \left(1 - e^{-t/\tau}\right) \longrightarrow (1)$$

The pulse height V at the output related to the amount of charge Q collected at the output through the relation

$$V = \frac{Q}{C} \longrightarrow (2)$$

C - Capacity of the o/p point.

Energy dissipated in the phosphor $= E_t A$.

This energy is converted within efficiency C_{Ip} into photons of average energy E_p

$$n_p = E_t A \, C_{Ip} / E_p.$$

The Resolution R of a scintillation counter can be defined as

(14)

$$\frac{1}{R} = \frac{\overline{Q^2} - (\overline{Q}^2)}{(\overline{Q})^2}$$

Where $\overline{Q}$ is the mean charge collected at the anode and $\overline{Q^2}$ is the mean squared charge collected at the anode.

The scintillation counter has a much higher counting efficiency for gamma rays due to the greater amount of energy dissipation by the gamma rays. Because of their extremely rapid response scintillation counters are having many applications in atomic energy field, cosmic rays study, for detection of mesons and other unstable particles of very high energy..

B is the magnetic field strength $B = 1$ T,

$$\upsilon = \frac{(1.6\times10^{-19}C)\times(1T)\times(0.5m)}{1.67\times10^{-27}kg}$$

$\upsilon \approx 4.8 \times 10^6$ m/s

Now, substitute υ into the frequency equation:

$$f = \frac{\upsilon}{2\pi R} = \frac{4.8\times10^6}{2\pi\times0.5}$$

$f \approx 1.53 \times 10^6$ Hz

30. A proton is accelerated by a cyclotron with a magnetic field of B = 2T and a radius of R = 1.2 m. Find the kinetic energy of the proton when it completes 10 turns in the cyclotron.

Sol. We have $KE = \frac{q^2B^2r^2}{2m}$

We have

m is the mass of the proton,

υ is the velocity of the proton,

q is the charge of the proton,

B is the magnetic field.

Substitute the known values:

$q = 1.6 \times 10^{-19}$ C,

$B = 2T$,

$R = 1.2$ m,

$m = 1.67 \times 10^{-27}$ kg.

$$K.E = \frac{1}{2}\times\frac{(1.6\times10^{-19})^2\times(2)^2\times(1.2)^2}{1.67\times10^{-27}}$$

$K \approx 2.3 \times 10^{-13}$ J

❖❖❖❖❖

UNIT V

APPLICATIONS OF NUCLEAR AND PARTICLE PHYSICS

ESSAY QUESTIONS

1. Explain the applications of Nuclear and Particle Physics in Radiation therapy.

Ans. Nuclear and particle physics play a crucial role in the field of radiation therapy, particularly in the treatment of cancer. These fields are applied to deliver targeted radiation to tumours, minimizing damage to surrounding healthy tissues.

Radiation Therapy: Radiation therapy involves using high-energy radiation to damage the DNA of cancer cells, which reduces their ability to divide and grow. The main forms of radiation used are X-rays (photon therapy), Electron beams, Nuclei of hydrogen (protons) or heavier atoms (such as carbon).

Nuclear Physics in Radiation Therapy:

Nuclear physics deals with the behaviour and interactions of atomic nuclei. In radiation therapy, nuclear processes are connected to produce radiation:

Radioactive isotopes: Certain radioactive isotopes, such as Cobalt-60 and Iridium-192, are used in brachytherapy, a type of internal radiation therapy. These isotopes emit gamma rays, which are used to treat localized tumours.

Medical Imaging (PET and SPECT):

These imaging techniques use radioactive isotopes to detect and monitor cancer. In Positron Emission Tomography (PET), for instance, a radioactive tracer like fluorodeoxyglucose (FDG) is injected into the body. Cancerous tissues tend to absorb more of this tracer, allowing doctors to identify and target tumours with greater precision.

Radiation dosimetry: In delivering the right dose of radiation to the tumour while sparing normal tissues dosimeters are used. They measure the dose of radiation.

Particle Physics in Radiation Therapy:

Particle physics focuses on the interactions of subatomic particles. In radiation therapy, particle accelerators are used to generate high-energy particles for treatment:

Proton Therapy: Protons are subatomic particles that carry a positive charge. Proton therapy, an advanced form of radiation therapy, uses proton beams generated by a cyclotron or synchrotron accelerator. Proton therapy is precise because protons deposit most of their energy at a specific depth, minimizing damage to healthy tissues beyond the tumour.

Heavy Ion Therapy: Similar to proton therapy, heavy ions (e.g., carbon ions) are accelerated and used to treat tumours. Heavy ions have higher mass and charge than protons, and their radiation dose distribution is even more concentrated. This makes them particularly useful for treating deep or radioresistant tumours.

Radiation interactions with matter: Particle accelerators also help scientists and doctors understand how radiation interacts with biological tissues. This knowledge allows the design of more effective treatments and the optimization of treatment planning.

Techniques in Particle Radiation Therapy:

Cyclotrons and Synchrotrons: These particle accelerators are central to proton and heavy ion therapies. They accelerate protons or heavier ions to high energies, directing them toward the tumour.

Bragg Peak: This concept is vital in particle therapy. As protons or heavy ions travel through tissues, they deposit energy gradually. The Bragg peak is the point where most of the energy is released, allowing for the highly localized delivery of radiation to tumours. This reduces the damage to surrounding healthy tissue compared to conventional photon radiation.

Intensity-Modulated Proton Therapy (IMPT):

IMPT allows for even more precise targeting of tumours, adjusting the intensity of proton beams to match the shape of the tumour.

Advantages of Particle Therapy:

1. Precision: Particle therapy (especially proton and heavy ion therapy) can treat tumours with great precision, sparing adjacent healthy tissues and critical organs.

2. Reduced side effects: Because of the precise dose distribution, patients often experience fewer side effects compared to traditional X-ray therapy.

3. Deep tumours: Particle therapy is particularly useful for deep tumors or those in close proximity to sensitive structures, such as the brain or spine, where conventional radiation might cause more harm.

4. Treatment of radioresistant tumours: Heavy ions and protons can be more effective at destroying certain types of cancer cells, particularly those that are resistant to standard X-ray treatments.

2. Explain the applications of Nuclear and Particle Physics in medical imaging techniques.

Ans. Nuclear and particle physics have enabled the development of a range of imaging techniques that enhance the accuracy, specificity, and effectiveness of medical diagnoses and treatments. Techniques like PET, SPECT, CT, and MRI provide crucial insights into the structure and function of the body, contributing to early disease detection, treatment planning, and improved patient outcomes.

1. Positron Emission Tomography (PET):

Working: PET is a nuclear medicine imaging technique that uses radioactive tracers, which emit positrons, to produce detailed images of the body's internal structures and functions. A small amount of a radiotracer (often a form of glucose or other molecules labelled with a positron-emitting isotope like fluorine-18) is injected into the body.

Physics principle: The radioactive isotope decays, emitting a positron. The positron interacts with an electron in the body, leading to annihilation and the production of two gamma rays travelling in opposite directions. These gamma rays are detected by a PET scanner.

Applications: PET is particularly useful for detecting cancer (as tumors often have high metabolic activity), assessing heart function, monitoring brain activity, and guiding surgeries.

2. Single Photon Emission Computed Tomography (SPECT):

Working: SPECT is similar to PET but uses gamma rays emitted by a radiotracer, which is injected into the patient. The radiotracer is typically a compound that binds to specific tissues (e.g., bone, heart, or brain).

Physics principle: A gamma camera detects the emitted gamma rays from the tracer as they pass through the body, and a computer processes the data to generate 3D images of the organ or tissue of interest.

Applications: SPECT is commonly used to assess blood flow to the heart, brain activity, and bone scans for detecting conditions such as osteoporosis or infections.

3. X-ray Imaging:

Working: X-ray imaging involves passing high-energy X-rays (a form of electromagnetic radiation) through the body. The X-rays are absorbed to varying degrees by different tissues, with denser tissues (e.g., bones) absorbing more and appearing white on the X-ray image.

Physics principle: X-rays interact with matter and are either absorbed or scattered depending on the material's density and atomic number. The resulting images are created based on the differential absorption.

Applications: X-rays are used in routine diagnostic imaging such as chest X-rays, dental X-rays, mammograms, and CT scans, providing detailed information about bones, organs, and soft tissues.

4. Computed Tomography (CT) Scanning:

Working: A CT scan combines multiple X-ray images taken from different angles and uses computational techniques to reconstruct cross-sectional images of the body.

Physics principle: CT uses X-ray technology, but the X-ray source and detectors rotate around the patient, capturing numerous images from various angles. These are then reconstructed using advanced algorithms to form 3D images.

Applications: CT scans are invaluable for detecting and diagnosing a wide range of conditions, including trauma, cancers, infections, and vascular issues, providing highly detailed cross-sectional images of organs and tissues.

5. Magnetic Resonance Imaging (MRI):

Working: MRI does not involve nuclear reactions but leverages the principles of nuclear physics related to the magnetic properties of atomic nuclei. In MRI, hydrogen nuclei (protons) in the body are

aligned by a strong magnetic field. A pulse of radiofrequency energy excites these protons, and when they return to their original state, they emit signals that are detected and used to create detailed images.

Physics principle: MRI relies on the magnetic properties of nuclei (primarily hydrogen in water and fat), which behave like tiny magnets in a magnetic field. The variation in the resonance frequencies of protons allows detailed imaging of tissues with high spatial resolution.

Applications: MRI is crucial for imaging soft tissues, such as the brain, spinal cord, muscles, and organs, and is used to diagnose conditions like tumours, neurological disorders, and musculoskeletal injuries.

6. Gamma Knife Radio surgery:

Working: Gamma Knife is a form of stereotactic radiosurgery that uses targeted gamma radiation (often from Cobalt-60 isotopes) to treat brain tumors, arteriovenous malformations, and other brain disorders without invasive surgery.

Physics principle: A helmet-like device contains multiple radiation sources that focus beams of gamma radiation on a precise point in the brain, allowing for the delivery of high doses of radiation to a very localized area, while minimizing damage to surrounding tissues.

Applications: Gamma Knife radio surgery is primarily used in treating brain tumors, arteriovenous malformations, and functional disorders like trigeminal neuralgia.

7. Radiation Therapy Planning and Image-Guided Radiotherapy (IGRT):

Working: In radiation therapy, high-energy radiation is used to treat cancerous tumours. Imaging techniques like CT, MRI, and PET scans are used to precisely locate tumours, helping to design radiation treatment plans and guide the delivery of radiation.

Physics principle: Particle physics principles, such as the interaction of high-energy radiation with matter, allow for the accurate targeting of tumours while minimizing exposure to healthy tissues.

Applications: Radiation therapy, guided by imaging, is essential in treating cancers, where precise delivery of radiation is necessary to destroy tumour cells while sparing surrounding healthy tissues.

3. Explain the role of Particle Physics in High-Energy Astrophysics.

Ans. Particle physics plays an essential role in understanding the extreme environments in high-energy astrophysics. In the study of cosmic rays, black holes, or dark matter, the intersection between these two fields provides explanation for the most fundamental questions about the nature of matter, energy, and the universe's evolution. High-energy astrophysics and particle physics together shape our understanding of the cosmos.

Particle Physics and High-Energy Astrophysics Intersect in following areas:

1. Cosmic Rays:

Nature: Cosmic rays are high-energy particles that travel through space and strike Earth's atmosphere. They primarily consist of protons, atomic nuclei, and electrons, with energies far exceeding those achievable by human-made accelerators.

Role of Particle Physics: Particle physics experiments, such as those performed at large particle detectors (e.g., the Pierre Auger Observatory, Ice Cube Neutrino Observatory), analyse cosmic rays to study their composition, energy spectrum, and origin. High-energy cosmic rays can help us understand the processes occurring in extremely energetic astrophysical objects like supernovae, active galactic nuclei, and gamma-ray bursts.

2. Gamma-Ray Astronomy:

Nature: Gamma rays are the most energetic form of electromagnetic radiation. They are produced in extremely high-energy processes such as black hole accretion disks, neutron star mergers, and supernova explosions.

Role of Particle Physics: Particle physics helps to understand the mechanisms of high-energy photon production. For instance, high-energy gamma rays are often the result of electron-positron annihilation or interactions between cosmic protons and the interstellar medium. Astrophysical instruments like the Fermi Gamma-ray Space Telescope detect gamma rays, and particle physicists analyse the interactions to gain insights into these cosmic sources.

3. Neutrinos and High-Energy Neutrino Astronomy:

Nature: Neutrinos are extremely light, weakly interacting particles. High-energy neutrinos are produced in astrophysical environments like supernovae, black holes, and active galactic nuclei (AGN).

Role of Particle Physics: Neutrinos are crucial for understanding the processes occurring in these distant objects. The detection of high-energy neutrinos helps study particle physics phenomena that occur under extreme conditions. For example, the Ice Cube Neutrino Observatory detects neutrinos originating from astrophysical sources, providing information on supernovae, AGNs, and cosmic ray interactions.

Neutrino Flavour Oscillations: The discovery of neutrino oscillations (the phenomenon where neutrinos change flavour as they travel) has deep implications for both particle physics and astrophysics, providing a window into the properties of neutrinos in the universe.

4. Black Hole Physics:

Nature: Black holes are regions of space-time where gravity is so strong that not even light can escape. The accretion disk around black holes can accelerate particles to near light speed, creating high-energy phenomena.

Role of Particle Physics: The behaviour of matter near black holes can be studied by examining the gamma rays, X-rays, and neutrinos emitted. Particle physics helps model the interactions of high-energy particles in accretion disks, jets, and near event horizons, as well as the Hawking radiation (predicted radiation emitted by black holes due to quantum effects).

5. Dark Matter and Dark Energy:

Nature: Dark matter and dark energy make up about 95% of the universe's total energy content but are not directly detectable by traditional means (like visible light). Dark matter is thought to be composed of particles that do not interact via electromagnetic force, making it "dark" to telescopes.

Role of Particle Physics: Understanding dark matter is one of the biggest challenges in both particle physics and astrophysics. The search for Weakly Interacting Massive Particles (WIMPs) or other potential dark matter candidates is central to high-energy astrophysics. Experiments like the Large Hadron Collider (LHC) and underground detectors (e.g., XENON1T, LUX-ZEPLIN) aim to detect dark matter particles directly or indirectly by observing their effects on regular matter.

Dark Energy: Dark energy is believed to be responsible for the accelerated expansion of the universe. Particle physics models, including those related to the "cosmological constant" or "quintessence" theories, attempt to explain the nature of dark energy through its effect on cosmic acceleration and large-scale structure formation.

6. Inflation and the Early Universe:

Nature: The theory of cosmic inflation proposes that the universe underwent a period of extremely rapid expansion during the first fraction of a second after the Big Bang. During this period, quantum fluctuations in the early universe were stretched to macroscopic scales, influencing the formation of the universe's structure.

Role of Particle Physics: Particle physics plays a crucial role in understanding inflation by proposing models for the fundamental particles and fields that drove inflation. For instance, scalar fields such as the inflation field are key to the inflationary theory. Observations of the cosmic microwave background (CMB) radiation, particularly the anisotropies (small temperature fluctuations), provide clues about the physics of the early universe, which are tested using high-energy particle physics models.

7. High-Energy Astrophysical Jets:

Nature: Jets of particles are often ejected from the poles of black holes, neutron stars, or other compact objects, sometimes at speeds close to the speed of light.

Role of Particle Physics: The processes that accelerate particles in these jets are similar to those studied in particle accelerators, involving magnetic fields, plasma physics, and relativistic effects. Particle physics helps explain the mechanisms of particle acceleration (e.g., Fermi acceleration) and particle interactions in these extreme environments.

8. Gravitational Waves and High-Energy Astrophysics:

Nature: Gravitational waves are ripples in space time caused by violent astrophysical events, like merging black holes or neutron stars.

Role of Particle Physics: High-energy astrophysical events that produce gravitational waves also produce other fundamental particles, such as gamma rays, neutrinos, or heavy ions. Understanding these particles helps in the interpretation of gravitational wave events. The study of gravitational wave astronomy opens new windows into both general relativity and particle physics, particularly in extreme environments.

4. Explain the concept of Nuclear Reactors and Power Generation.

Ans. Nuclear reactors play a pivotal role in the generation of electricity through nuclear fission, a process that releases a vast amount of energy from the nucleus of atoms. In a nuclear reactor, the controlled splitting of atoms (usually uranium or plutonium) generates heat, which is then used to produce steam that drives turbines connected to generators. This is the fundamental process behind nuclear power plants.

Working of nuclear reactors and their role in power generation:

Components of a Nuclear Reactor: A nuclear reactor consists of several key components that work together to generate power:

a. Fuel:

Uranium or Plutonium: The most common nuclear fuel used in reactors is Uranium-235, though Lutonium-239 is also used in some reactors. Natural Uranium consists mostly of Uranium-238, which is not readily fissionable, so it must be enriched (increasing the proportion of Uranium-235) for use in most reactors.

b. Reactor Core:

The reactor core contains the nuclear fuel, where the fission reactions take place. It is designed to withstand high temperatures and pressures and to contain the radiation produced.

c. Coolant:

The coolant (often water) circulates through the reactor to transfer the heat generated by fission to the steam generators. In many reactors, the coolant also acts as the moderator, slowing down neutrons to sustain the chain reaction.

In pressurized water reactors (PWRs), the coolant is kept under high pressure to prevent it from boiling, even at temperatures as high as 320°C.

d. Steam Generator:

In most reactors, the coolant heats water in a separate loop to generate steam. This steam drives a turbine connected to a generator, producing electricity.

e. Turbine and Generator:

The steam turbine converts the thermal energy of steam into mechanical energy, which is used to drive a generator. The generator then converts mechanical energy into electrical energy, which is sent to the grid.

4. Containment Structure:

A containment structure is a thick concrete and steel shell that surrounds the reactor, designed to contain any radioactive materials that may be released in the event of an accident, and to prevent the escape of radiation into the environment.

Nuclear Power Generation Process:

1. Fuel Loading: Uranium fuel is loaded into the reactor core.

2. Chain Reaction Initiation: Neutrons are introduced to initiate the fission chain reaction.

3. Heat Production: As the fission reactions occur, large amounts of heat are produced.

4. Cooling: The coolant (water or gas) removes the heat from the core and transfers it to a steam generator.

5. Steam Generation: The coolant transfers heat to a secondary loop, producing steam.

6. Turbine and Power Generation: The steam drives a turbine that spins a generator, producing electricity.

7. Cooling of Steam: After passing through the turbine, the steam is cooled, usually by a cooling tower or nearby body of water, and is condensed back into water.

8. Electricity Distribution: The generated electricity is fed into the power grid.

SHORT ANSWER QUESTIONS

5. Write a short note on the applications of Nuclear and Particle Physics in Radiation therapy.

Ans. Nuclear and particle physics have revolutionized radiation therapy, enabling more precise, effective, and safer treatments for cancer patients. By leveraging the principles of nuclear reactions and subatomic particle interactions, modern therapies like proton and heavy ion therapy offer a promising future in cancer treatment. Advances in particle accelerator technology and radiation delivery systems continue to enhance the accuracy and efficiency of these therapies, benefiting patients worldwide.

Nuclear and particle physics have significant applications in radiation therapy and medical imaging techniques, particularly in the

diagnosis and treatment of various diseases, including cancer. These applications utilize the properties of radiation, subatomic particles, and nuclear reactions to precisely target disease tissues or provide diagnostic images for medical evaluation.

6. Mention the applications of Nuclear and Particle Physics in medical imaging techniques.

Ans. 1. Positron Emission Tomography (PET):

Applications: PET is particularly useful for detecting cancer (as tumors often have high metabolic activity), assessing heart function, monitoring brain activity, and guiding surgeries.

2. Single Photon Emission Computed Tomography (SPECT):

Applications: SPECT is commonly used to assess blood flow to the heart, brain activity, and bone scans for detecting conditions such as osteoporosis or infections.

3. X-ray Imaging:

Applications: X-rays are used in routine diagnostic imaging such as chest X-rays, dental X-rays, mammograms, and CT scans, providing detailed information about bones, organs, and soft tissues.

4. Computed Tomography (CT) Scanning:

Applications: CT scans are invaluable for detecting and diagnosing a wide range of conditions, including trauma, cancers, infections, and vascular issues, providing highly detailed cross-sectional images of organs and tissues.

5. Magnetic Resonance Imaging (MRI):

Applications: MRI is crucial for imaging soft tissues, such as the brain, spinal cord, muscles, and organs, and is used to diagnose conditions like tumours, neurological disorders, and musculoskeletal injuries.

6. Gamma Knife Radio surgery:

Applications: Gamma Knife radio surgery is primarily used in treating brain tumors, arteriovenous malformations, and functional disorders like trigeminal neuralgia.

7. Radiation Therapy Planning and Image-Guided Radiotherapy (IGRT):

Applications: Radiation therapy, guided by imaging, is essential in treating cancers, where precise delivery of radiation is necessary to destroy tumour cells while sparing surrounding healthy tissues.

7. Mention the Tools and Techniques Used in High-Energy Astrophysics.

Ans. Particle Detectors: Instruments such as detectors in space (e.g., Fermi Telescope) and ground-based observatories (e.g., IceCube) capture high-energy particles and photons. These detectors analyze cosmic ray particles, neutrinos, and gamma rays.

Particle Accelerators: Large-scale particle accelerators, like the LHC, are used to recreate conditions similar to those in the early universe or to study interactions that might occur in astrophysical phenomena.

Telescopes and Observatories: Gamma-ray telescopes, X-ray observatories, and gravitational wave detectors observe high-energy cosmic events and help researchers test particle physics theories.

8. Briefly explain the basic Principles of Nuclear Power Generation.

Ans. The basic Principles of Nuclear Power Generation are,

1. Nuclear Fission:

Process: Nuclear fission is the process in which the nucleus of a heavy atom, such as Uranium-235 or Plutonium-239, splits into two smaller nuclei, along with a few neutrons and a significant amount of energy in the form of heat. This is the key reaction that drives nuclear power generation.

Chain Reaction: When the nucleus of Uranium or Plutonium absorbs a neutron, it becomes unstable and splits, releasing energy and additional neutrons. These neutrons can further split more nuclei, creating a chain reaction. The reactor must maintain a controlled rate of fission to avoid dangerous overheating.

2. Control of the Chain Reaction:

Control Rods: To prevent the reaction from becoming uncontrollable, nuclear reactors use control rods, typically made from materials like boron or cadmium, which absorb neutrons. By adjusting the position of the control rods inside the reactor, operators can control the rate of fission.

Moderators: A moderator (usually water or graphite) is used in most reactors to slow down the neutrons produced by fission. Slower neutrons are more likely to induce further fission reactions in uranium, ensuring the chain reaction continues at a steady rate.

9. Briefly mention the types of Nuclear Reactors.

Ans. There are several different types of nuclear reactors, each with specific designs and advantages:

1. Pressurized Water Reactors (PWR): The PWR is the most widely used reactor type worldwide. It uses water under high pressure as both the coolant and the moderator.

Operation: The primary loop circulates water under pressure through the reactor core to transfer heat. A secondary loop uses the heat from the primary loop to generate steam, which drives the turbine.

2. Boiling Water Reactors (BWR): In a BWR, the water in the reactor core boils to produce steam directly within the reactor vessel. The steam is then sent to the turbine.

Fewer Loops: Unlike PWRs, BWRs have a simpler design since they use only one loop for both cooling and steam production.

3. CANDU (Canadian Deuterium Uranium) Reactors: CANDU reactors use heavy water (deuterium oxide, D_2O) as a moderator and coolant. This allows the reactor to use natural, unenriched uranium as fuel, reducing the cost of fuel.

Advantages: The ability to use natural uranium as fuel and the flexibility to refuel the reactor while it is operating are key features of CANDU reactors.

4. Fast Breeder Reactors (FBR): FBRs are designed to produce more fuel than they consume. They use fast neutrons (without slowing them down) to convert fertile Uranium-238 into fissile Plutonium-239, which can then be used as fuel.

High Efficiency: FBRs can make more efficient use of Uranium and potentially extend the life of nuclear fuel supplies.

5. Molten Salt Reactors (MSR): MSRs use molten salt as both the coolant and the fuel. This design allows for higher operating temperatures and potentially safer reactor operations, with the added benefit of using thorium or uranium as fuel.

Safety: MSRs have inherent safety features due to their ability to operate at atmospheric pressure and the design that allows for the reactor to "drain" in case of an emergency.

10. Give the advantages and drawbacks of Nuclear Power.

Ans. Advantages:

1. **Low Greenhouse Gas Emissions:** Nuclear power is a low-carbon energy source, producing significantly fewer greenhouse gases compared to fossil fuels like coal or natural gas.

2. **High Energy Density:** The amount of energy produced by nuclear fuel is orders of magnitude higher than that from chemical fuels (like coal or oil), meaning nuclear reactors require much less fuel.

3. **Reliable and Stable Energy Source:** Nuclear power plants provide base load power* meaning they can operate continuously for long periods (typically 18-24 months) before needing to refuel.

Drawbacks:

1. **Radioactive Waste:** One of the main challenges of nuclear energy is the long-lived radioactive waste it generates, which requires secure storage and management for thousands of years.

2. **Safety:** While modern reactors have many safety features, the potential for accidents (such as Chernobyl or Fukushima) raises concerns about the safety of nuclear energy.

3. **High Initial Costs:** The construction of nuclear power plants is capital-intensive and requires significant investment, although operational costs are relatively low.

4. **Nuclear Proliferation:** The technology for enriching uranium or reprocessing plutonium raises concerns about the potential for nuclear weapons development.

❖❖❖❖❖

MODEL PAPERS

MODEL PAPER - I

Time : 3 Hours] W.E.F from 2023-2024 [Max. : 75 Marks

SECTION – A (5 × 5 = 25 Marks)

Answer any FIVE of the following questions.

1. Explain the terms Mass defect, Binding energy. [Q.No. 7, Pg No. 13]
2. Explain the concept of magic numbers. [Q.No. 9, Pg No. 14]
3. Write a short note on Isospin. [Q.No. 15, Pg No. 47]
4. Mention the applications of Nuclear Reactions. [Q.No. 8, Pg No. 67]
5. Explain the energy distribution in Beta Decay. [Q.No. 8, Pg No. 94]
6. A cyclotron with dees of radius 2 meters has a magnetic field of 0.75 Weber/m^2. Calculate the maximum energies to which a proton can be accelerated. [Q.No. 27, Pg No. 104]
7. Write a short note on the applications of Nuclear and Particle Physics in Radiation therapy. [Q.No. 5, Pg No. 116]
8. Give the advantages and drawbacks of Nuclear Power. [Q.No. 10, Pg No. 120]

SECTION – B (5 × 10 = 50 Marks)

Answer any FIVE of the following questions.

9. a) Explain the general Properties of Nuclei. [Q.No. 1, Pg No. 1]

(OR)

b) Explain Semi empirical mass formula. [Q.No. 6, Pg No. 10]

Major II

Introduction to Nuclear and particle physics.

UNIT- I

Introduction to Nuclear physics.

→ Nuclear Structure:-

From Rutherford α-particle scattering, the atom of any element consists of central core called Nucleus, and electrons moving around it. The nucleus consists of two particles the proton and neutron. And their masses are

Mass of proton $= 1.67261 \times 10^{-27}$ kg

Mass of Neutron $= 1.67492 \times 10^{-27}$ kg.

Both protons and neutrons together in the nucleus are called Nucleons. The no of protons in the nucleus is called the atomic no and the sum of protons and neutrons is called as mass no.

(2) General properties of Nuclei :-

1) **Nuclear Size :-**

Nucleus diameter can be measured in a no of ways 1) Scattering of fast protons & neutrons and 2) by scattering of high energy electrons.

Rutherford found this distance is of the order of 10^{-14} m. The radius of an atomic nucleus can be determined from the following relation

$$\boxed{r = r_0 A^{1/3}}$$

where r_0 - linear const and $= 1.4 \times 10^{-15}$ m.

Eg:- Carbon (A = 12) $r = 1.4 \times 10^{-15} \times (12)^{1/3}$

$= 3.21 \times 10^{-15}$ m.

2) **Nuclear Mass :-**

The mass of the nucleus is the sum of the masses of neutrons and protons contained it. It expressed in terms of atomic mass unit (a.m.u)

1 amu $= 1.66 \times 10^{-27}$ kg $= 931.48$ MeV.

$_6C^{12}$ — mass - 12 amu. , A = 12, Z = 6.

3) **Nuclear charge density :-**

The charge on the nucleus is due to protons contained in it. charge of each proton is 1.6×10^{-19} columb. The density of the nucleus can be calculated as follo

Volume of the nucleus $= \frac{4}{3}\pi r^3$

r - radius of the nucleus

$r = 1.5 \times 10^{-15} A^{1/3}$ m. (3)

Volume of the Nucleus $= \frac{4}{3}\pi (1.5\times10^{-15})^3 A$. m^3

$= 14.15 \times 10^{-45} A$ meter^3

Mass of the nucleus = A× mass of proton.

$= 1.673 \times 10^{-27} A$ kg

He^4_2 - mass no. 2 - atomic no. $_Z^A He$

$\therefore$ Density of Nucleus $= \dfrac{1.673 \times 10^{-27}}{14.15 \times 10^{-45}}$ kg/m³

$= 1.18 \times 10^{17}$ kg/m³.

4) Magnetic dipole Moment (μ):-

A charged particle moving in a closed path produces a mag field. The magnetic field at a large distance may be regarded due to a magnetic dipole located at the current loop.

The Spining electron has an magnetic dipole moment of I.

$\mu_e = \dfrac{e\hbar}{2m_e}$ e - charge, m_e - mass of electron.

According to Dirac Theory

$$\mu_N = \frac{e(h/2\pi)}{2m_p} = \frac{e\hbar}{2m_p}$$

where m_p - proton mass = $1830 m_e$

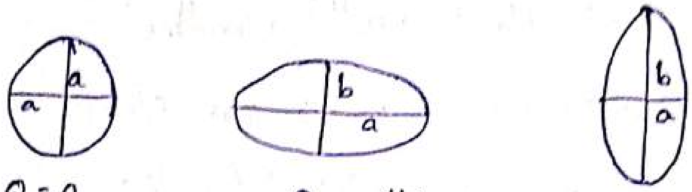
μ_N = nuclear magneton

$\mu_N = 1.9128 \mu_N$,

5) Electric (Quadrupole) moment (Q):-

The Shape of the nucleus is not Spherical but it is an ellipsoid of revolution.

4) The ~~a~~ deviation from the spherical symmetry is expressed in terms of a quantity known as electric quadrupole moment.



$Q=0$ $\quad$ $Q = -ve$ $\quad$ $Q = +ve.$

The electric quadrupole moment is given by

$$Q = \frac{2}{5} Ze \left(b^2 - a^2\right)$$

Z - atomic no and Ze - total charge on the nucleus.

$Q = 0$ for spherical shaped nucleus.

$Q = -ve$ for ellipsoid $(a > b)$

$Q = +ve$ for ellipsoid $(b > a)$.

6) parity and Angular Momentum:-

The total angular momentum of a nucleus is formed from the sum of the individual constituents, angular momentum l and spin angular momentum s, The symbol is given to the nuclear angular momentum is I. Then

$$I = \sum_{i=1}^{A} (l_i + s_i)$$

$$\& \; I^2 = \hbar^2 I (I+1), \quad I = 0, \tfrac{1}{2}, 1, \tfrac{3}{2} ---$$

$$m_I = -I \le m_I < I \qquad \Delta m_I - \text{integral}.$$

The parity is associated with a quantum no of ± 1. The parity operator acting on the composite nuclear wave function $\psi(\bar{r}, A, Z)$

$$\Pi \psi(\bar{r}, A, Z) = +\psi(-r, A, Z)$$

The + sign associated with even parity and − sign odd parity.

→ Mass Defect:-

The mass of a stable nucleus is less than sum of masses of the constituents nucleons. (P & N).

Real nuclear mass, $M < Zm_p + Nm_N$

The difference between the sum of the masses of nucleons constituting the nucleus and real mass of nucleus is known as mass defect.

$$\text{Mass defect } \Delta m = [Zm_p + Nm_n] - M.$$

$$\Delta m = [Zm_p + (A-Z)m_n] - M.$$

m_p - mass of proton, m_n - mass of neutron

M - Mass of nucleus Z - no of protons.

A - mass no.

and $N = (A-Z)$ no of neutrons.

→ Binding Energy of Nucleus:-

The neutrons and protons within a nucleus are held together by strong attractive forces among the nucleons. To break a nucleus into its constituents, certain amount of energy is required against the attractive forces. This energy is known as binding energy of the nucleus.

The binding energy is defined as the energy required to decomposed a nucleus into its constituents particles.

(6) The difference is measured mass M and the mass number A is called as mass defect.

$$\text{mass defect } \Delta m = (M-A).$$

This mass Δm is converted into an amount of energy $\Delta E = (\Delta m)c^2$. This energy is called as binding energy of the nucleus.

If M is the mass of a nucleus having Z protons and N neutrons then

$$B.E = \left[(Z m_p + N m_n) - M\right]c^2 \longrightarrow (1)$$

When B.E < 0, the nucleus is unstable.

If A is the mass no in the nucleus

$$B.E = \left[Z m_p + (A-Z) m_n - M\right]c^2$$

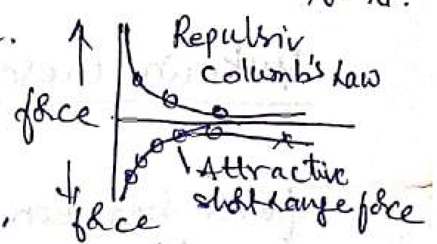
Nucl

→ Nuclear forces; characteristics of nuclear forces:-

The nucleus consists of ~~p~~ protons and neutrons. There must be some strong attractive forces between protons and neutrons and this force cant be gravitational force because they are ~~&~~ much smaller, and also not electrical force in nature because the strong repulsive forces b/w protons will lead to disruption of nucleus.

Actually these forces are short range attractive forces known as nuclear forces. The nuclear forces have the following properties.

1. These forces are attractive forces between P–N, P–P, N–N.
2. These forces are strongest in nature.
3. They are spin dependent.
4. These forces are short range forces.
5. These forces are independent of charge.
6. In case of nuclear forces, each nucleon attracts those nucleons which are immediate neighbours



→ Yukawa's ~~Meas~~ Meson theory of Nuclear forces:-

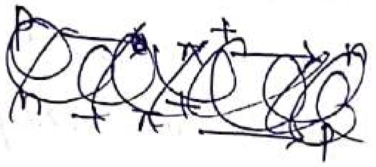
Yukawa in 1935 proposed a theory to explain the binding forces between neutrons and protons known as meson theory of nuclear forces.

According to this theory, there exists a new type of field between nucleons known as meson field. Yukawa pointed out that the nuclear forces ~~aitse~~ arise due to the continuous exchange of +ve and −ve particles (mesons) between nucleons. These particles are known as mesons because their mass being intermediate between electron and proton.

The exchange of meson between two nucleons develop an attractive force between them.

The meson have +ve & −ve charge which is equal to electronic charge. They are designated as π^+ and π^- meso.

When a π^+ meson jumps from a proton to a neutron the proton is converted into a neutron and vice versa.



$$p - \pi^+ \longrightarrow n$$
$$n + \pi^+ \longrightarrow p$$

When a π^- meson jumps from a neutron to proton, the neutron is converted into a proton and vice versa.

$$n - \pi^- \longrightarrow p$$
$$p + \pi^- \longrightarrow n.$$

Yukawa meson field Theory:- (No Need ?)

According to Yukawa, the ~~nuclear~~ nuclear forces between two nucleons is due to the interchange of particles called mesons.

As nuclear force field is short range, Yukawa represented the field by a potential ϕ given by

$$\phi = -\frac{e^{-ar}}{r}$$

which falls of rapidly at $r = \frac{1}{\alpha}$. This potential satisfies the field equation

$$\nabla^2\phi - \alpha^2\phi - \frac{1}{c^2}\frac{d^2\phi}{dt^2} = 0 \longrightarrow (1)$$

where $\nabla^2\phi = \frac{d^2\phi}{dx^2} + \frac{d^2\phi}{dy^2} + \frac{d^2\phi}{dz^2}$, α is a new universal const equal to 0.3 to $0.5 \times 10^{15}\, m^{-1}$ and c is the velocity of light.

The meson which is a quantum of nuclear force can be described by matter waves.

Let us consider a plane wave of the form

$$\phi = A\sin(Kx - \omega t) \longrightarrow (2)$$

where $K = \frac{2\pi}{\lambda}$, $\lambda = \frac{h}{mv} = \frac{h}{p}$, $\omega = 2\pi\nu$ and $\nu = \frac{E}{h}$.

from eq (2) we get

$$\frac{d^2\phi}{dx^2} = -K^2\phi = -\frac{4\pi^2}{\lambda^2}\phi$$

and

$$\frac{d^2\phi}{dt^2} = -\omega^2\phi = -4\pi^2\nu^2\phi$$

Substituting these values in eq (1) we get

$$-\frac{4\pi^2}{\lambda^2}\phi - \alpha^2\phi - \frac{1}{c^2}(-4\pi^2\nu^2\phi) = 0$$

$$\& \quad \frac{\nu^2}{c^2} = \frac{1}{\lambda^2} + \frac{\alpha^2}{4\pi^2} \longrightarrow (3)$$

The relativistic relation between energy E and momentum P is given by

$$E^2 = p^2c^2 + m_0^2c^4$$

where m_0 is the rest mass of meson. This expression can be written as

$$\left(\frac{E}{c}\right)^2 = p^2 + m_0^2c^2 \longrightarrow (4)$$

Now eq (3) in the form of Energy E can be written as

$$\frac{E^2}{h^2c^2} = \frac{p^2}{h^2} + \frac{\alpha^2}{4\pi^2}$$

$$\left(\frac{E}{c}\right)^2 = p^2 + \left(\frac{h\alpha}{2\pi}\right)^2 \longrightarrow (5)$$

Comparing relations (4) and (5) we see that the particle associated with the wave in question has a rest mass

$$m_0 = \left(\frac{h\alpha}{2\pi c}\right).$$

Substituting the value of α, h and c in this expression, get the rest mass m_0 which is 130 to 200 times the ele mass.

Thus predicting a new particle of mass Intermediate to the masses of electron and proton known as meson. Meson may be neutral & may carry either charge. This Yukawa particle was shown to be the pion (π-meson) discovered in 1947 in cosmic radiation.

→ ~~Nuclear Radiation Detectors~~:-

The phenomenon of spontaneous emission of radiations from radioactive substance is known as radioactivity. Eg:- uranium, polonium, radium, radon, ionium, thorium, actinium etc. The radioactivity is the property of the atom of the element.

Rutherford and his co-workers discovered that the radioactive substances are of two types. α-radiations and β-radiations and after discovered by villard as γ-radiations.

α-particles are doubly ionized helium atoms.
β-particles are electrons and
γ-radiations are electromagnetic waves.

properties:-

1. All the 3 radiations possess highly penetrating power
2. The radioactive element transformed into a new element which is again radioactive.
3. The emission of radiations is spontaneous.
4. The emission of radiation is prolonged process.

Natural radioactivity and
Artificial radioactivity

Nuclear Models:-

Liquid Drop Model:-

This model was proposed by Bohr:-

1) According to this model, the nucleus is similar to a small electrically charged liquid drop.
i.e Nucleus takes a spherical shape for its stability

2) The nucleons move within the spherical enclosure like molecules in a liquid drop.

3) The motion of neucleons within nucleus is a measure of nuclear temp as the molecular motion of molecules in liquid is the measure of its temp.

4) The ~~ne~~ nucleons always share among them the total energy of the nucleus.

*[So the model is ~~baled~~ based on the following assumptions

① The material of nucleus is incompressible and the density of all the nuclei is the same.

2) The forces in the nucleus consist of a ~~colomb~~ Coulomb force between protons and powerful attractive nuclear forces]*

The following are the analogies b/w liquid drop and a nucleus:-

① Both are spherical in nature

② The density is independent of its volume but, the nucleus is independent of the nucleus while the density of the liquid depends upon its type.

③ The molecules in liquid drop interact over short ranges and so it three for nucleons in nucleus.

④ As the surface tension force act on the surface of a drop similarly a potential barrier acts on the surface of nucleus.

(12) (5) When the temp of the molecules in a liquid drop is increased, evaporation of molecules takes place. Illy when the nucleons in the nucleus are subjected to external energy a compound nucleus is formed which emits nucleons. This process is known as nuclear fission.

Merits :-

Following are the merits of liquid drop model.

1) It has been sucessfully applied in describing nuclear reactions and explaining nuclear fission.

2) The calculation of atomic masses and binding energies can be done with good accuracy.

This model fails to explain other properties in particular the magic numbers.

→ Semi- Empirical Mass Formula:-

In 1935 Von Weizsacker expressed the atomic mass of a nucleide in terms of the series of binding energy correction terms with the main mass contribution from proton and neutron. The modified expression of the mass is known as semi emperical mass formula.

$$ {}_{Z}M^{A} = Z M_p + N M_n - E_b \longrightarrow (1) $$

where ${}_{Z}M^{A}$ = Atomic mass of the nuclide

Z = Number of protons. , N = Number of neutrons

M_p = Mass of the proton

M_n = Mass of the neutrons.

E_b = Binding energy expressed in mass unit.

The value of E_b is calculated empirically as made up of a number of collection terms given by (13)

$$E_b = E_v + E_s + E_c + E_r + E_p.$$

Where E_v, E_s, E_c, E_r, E_p are volume energy collection, Surface energy, coulomb energy, asymmetry energy and pairing energy respectively.

1) $\boxed{E_V \propto A}$ & $\boxed{E_V = a_v A}$

As the total binding energy of the nucleus is proportional to the total number of nucleons A in it.

2) $\boxed{E_S = -a_S A^{2/3}}$

The no of nucleons on the surface are proportional to surface area of the nucleus and there for to r^2. where $r = r_0 A^{1/3}$, So the no of nucleons on the surface are proportional to $A^{2/3}$, Surface ~~Tension~~ energy is first analogous to the surface Tension of liquid.

3) $E_c = -a_c \frac{Z(Z-1)}{A^{1/3}}$.

where the coulomb repulsion is proportional to electrostatic potential energy hence coulomb repulsion of binding energy E_c is above.

So $E_b = E_V + E_s + E_c$.

Then $E_b = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}}$.

The binding energy per nucleon

(14) $$\frac{E_b}{A} = a_v - \frac{a_s}{A^{1/3}} - a_c\frac{Z(Z-1)}{A^{4/3}}.$$

Let us consider the contribution of asymmetry energy and pairing energy. The contributions are as follows.

The decrease in binding energy due to neutron excess may be taken as inversely proportional to A. It has been shown that the contribution of asymmetric effect to the binding energy of the nucleus is given by

$$E_a = -a_a\frac{(A-2Z)^2}{A}$$

Nuclei containing even no of protons and odd no of neutrons & vice versa ~~have~~ have intermediate stability. This pairing effect changes the binding energy as shown below.

$$\text{pairing energy } E_p = a_p A^{-3/4}$$

The final expression for binding energy of a nucleus with atomic mass no. A and atomic number Z is

$$E_b = a_v A - a_s A^{2/3} - a_c\frac{Z(Z-1)}{A^{1/3}} - a_a\frac{(A-2Z)^2}{A} + a_p\left(\frac{1}{A^{3/4}}\right)$$

This is Semi empirical binding energy formula

$${}_Z M^A = ZM_p + (A-Z)M_n - a_v A + a_s A^{2/3} + a_c\frac{Z(Z-1)}{A^{1/3}} + a_a\frac{(A-2Z)^2}{A} \pm E_p$$

It is valid for mass no $A > 15$.

→ Nuclear Shell Model :-

The nuclei containing protons and neutrons number 2, 8, 20, 50, 82, 126 etc known as magic numbers & shell numbers are exceptionally stable as in case of electronic shell rule of 2, 8, 18 etc. This shows that there are definite energy shells in the nuclei.
The observations are

1. The nuclei for which z (no of protons) and $(A-z)$ (no of neutrons) are 2 $[{}_2He^4, z=2$ and $A-z=2]$, and 8 $[{}_8O^{16}; z=8$ and $A-z=8]$ are more stable than their neighbours.
2. The nuclei for which z & $A-z$ is magic number are specially stable. Eg

$${}_{28}Ni^{62}\ [z=28,\ A-z=34].$$

$${}_{38}Sr^{88}\ [z=38,\ A-z=50]$$

$${}_{50}Sn^{120}\ [z=50,\ A-z=70]$$

$${}_{58}Ce^{140}\ [z=58,\ A-z=82]$$

$${}_{82}Pb^{208}\ [z=82,\ A-z=126]$$

3. The electric quadrupole moments of magic no nuclei are very low compared to those other nuclei.

~~Based on these~~ the shell model is based on the following two assumptions

1) Each nucleon moves freely in a force field described by the potential, which is a function of radial distance from the centre of the system.

(16) 2) The energy levels & shells are filled according to pauli exclusion principle.

Consider a nucleon of mass M with angular momentum $\sqrt{l(l+1)}\hbar$ is moving in a potential $V(r)$. Assuming that $V(r)$ is independent of θ and ϕ. the Schroedinger wave equation can be written as

$$\frac{d^2}{dr^2}(rR) + \frac{2M}{\hbar^2}\left[E - V(r) - \frac{l(l+1)\hbar^2}{2Mr^2}\right](rR) = 0 \longrightarrow (1)$$

where R is the radial wave function and E is the energy eigen value.

Here the same quantum no. n, l, j, m_l result in shell model is an atomic model.
The square well potential is represented by

$$\left.\begin{aligned} V(r) &= -V_0 \quad \text{for } r \leq R \\ &= 0 \quad \text{for } r > R \end{aligned}\right\} \longrightarrow (2)$$

The harmonic oscillator potential is given by

$$V(r) = -V_0 + \tfrac{1}{2} k r^2 \longrightarrow (3)$$

If we combine the square well potential and harmonic oscillator potential the new form of the potential will be

$$\left.\begin{aligned} V(r) &= -V_0\left(1 - \frac{r^2}{R}\right) \quad \text{for } r \leq R \\ &= 0 \quad r > R \end{aligned}\right\} \longrightarrow (4)$$

Using the above potential in eq (1) and solving it we get all the nuclear magic numbers (shell structure except ~~magicum~~ magic number 28, In this way shell model of the nucleus in an attempt to account for the existance of magic numbers and certain other nuclear properties in terms of nucleon behaviour in a common force field. 17

ELEMENTARY PARTICLES AND INTERACTIONS

UNIT II

ESSAY QUESTIONS

1. Write a note on discovery of elementary particles.

Ans. The discovery and classification of elementary particles has evolved over more than a century. The classification and understanding of elementary particles have been based on both experimental observations and theoretical models.

Early Discoveries: The Beginning of Particle Physics

1. The Electron (1897): The first elementary particle to be discovered was the electron, identified by J.J. Thomson in 1897. It helped to establish the concept of subatomic particles.

Properties:

Charge: One unit negative (– 1)

Mass: Very small compared to atoms.

Significance:

The discovery of the electron led to the development of the field of atomic physics and set the stage for further discoveries in particle physics.

2. The Proton (1917): The proton was discovered by Rutherford in 1917 during experiments involving the scattering of alpha particles.

Properties:

Charge: One unit positive (+1)

Mass: About 1836 times the mass of an electron.

Significance:

The proton was the first known particle in the atomic nucleus, leading to the understanding of atomic structure.

3. The Neutron (1932): The neutron was discovered by James Chadwick in 1932.

Properties:

Charge: Neutral (no charge)

Mass: Similar to that of the proton.

Significance:

The neutron explained the missing mass in the atomic nucleus and led to the understanding of nuclear forces.

Discoveries after the development of Quantum Mechanics and the Standard Model:/

4. The Muon (1936): The Muon was discovered by Anderson and Seth Neddermeyer in 1936, during cosmic ray experiments.

Properties:

Charge: (– 1) like the electron.

Mass: About 200 times that of an electron.

Significance:

The Muon was initially thought to be a heavier electron but was later classified as a distinct particle, contributing to the understanding of particle families.

5. The Neutrinos (1956): The electron neutrino was confirmed in 1956 by Clyde Cowan and Frederick Reines.

Properties:

Charge: Neutral

Mass: Extremely small (very difficult to detect).

Significance:

Neutrinos are involved in weak interactions and were vital in completing our understanding of particle interactions.

The Quark Model and Hadrons:

6. Quarks (1964): The quark model was proposed by Murray Gell-Mann and George Zweig in 1964, based on the patterns of hadrons observed. Quarks are the fundamental particles that combine to form composite particles (hadrons). There are six types (flavors) of quarks:

Up (u), Down (d), Charm (c), Strange (s), Top (t), Bottom (b).

Properties:

Quarks have fractional electric charges (e.g., + 2/3 for up quarks and – 1/3 for down quarks).

Quarks interact via the strong force and are confined within hadrons.

Significance:

The quark model became a cornerstone of the Standard Model of particle physics, explaining the structure of hadrons (e.g., protons, neutrons).

The Standard Model and Fundamental Interactions:

7. The Photon (1905): Albert Einstein's explanation of the photoelectric effect in 1905 led to the realization that light can be thought of as discrete packets of energy, later called photons.

Properties:

Charge: Neutral

Mass: Zero

Spin: 1 (boson)

Significance:

The photon mediates the electromagnetic force and is central to quantum electrodynamics (QED).

8. The Gluon (1970s): The Gluon was predicted in the 1970s as the mediator of the strong nuclear force that holds quarks together inside hadrons. Gluons were indirectly confirmed through experiments in particle accelerators, but the exact particle was not directly observed until later.

Properties:

Charge: Neutral

Mass: Zero (but they carry color charge, unlike photons).

Spin: 1 (boson)

Significance:

Gluons are the force carriers for the strong interaction, a fundamental force in the Standard Model.

9. The W and Z Bosons (1983): The W and Z bosons were discovered at CERN in 1983.

Properties:

W Boson: Mediates the weak force, responsible for processes like beta decay.

Z Boson: Mediates the neutral weak force.

Both bosons have significant mass compared to the photon.

Significance:

Their discovery confirmed the weak nuclear force as part of the electroweak unification.

10. The Higgs Boson (2012): The Higgs boson was discovered at CERN in 2012, after being predicted by the Standard Model in the 1960s by Peter Higgs and others.

Properties:

Charge: Neutral

Mass: Heavy (compared to other elementary particles)

Spin: 0 (scalar boson)

Significance:

The Higgs boson is associated with the Higgs field, which gives mass to other elementary particles. Its discovery was a major milestone in confirming the Standard Model of particle physics.

The elementary particles discovered up until today are classified into the following categories based on their interactions and properties:

1. **Fermions (Matter Particles):** Quarks (e.g., up, down, charm, strange, top, bottom), Leptons (e.g., electron, muon, tau, neutrinos)
2. **Bosons (Force-Carrying Particles):** Photon (mediates electromagnetic force), Gluon (mediates strong force), W and Z Bosons (mediate weak force), Higgs Boson (gives mass to particles)
3. **Hypothetical Particles:** Graviton (hypothetical particle that would mediate gravitational force, still unobserved).

Major 11

(1)

Introduction to Nuclear and particle physics.

Unit - II

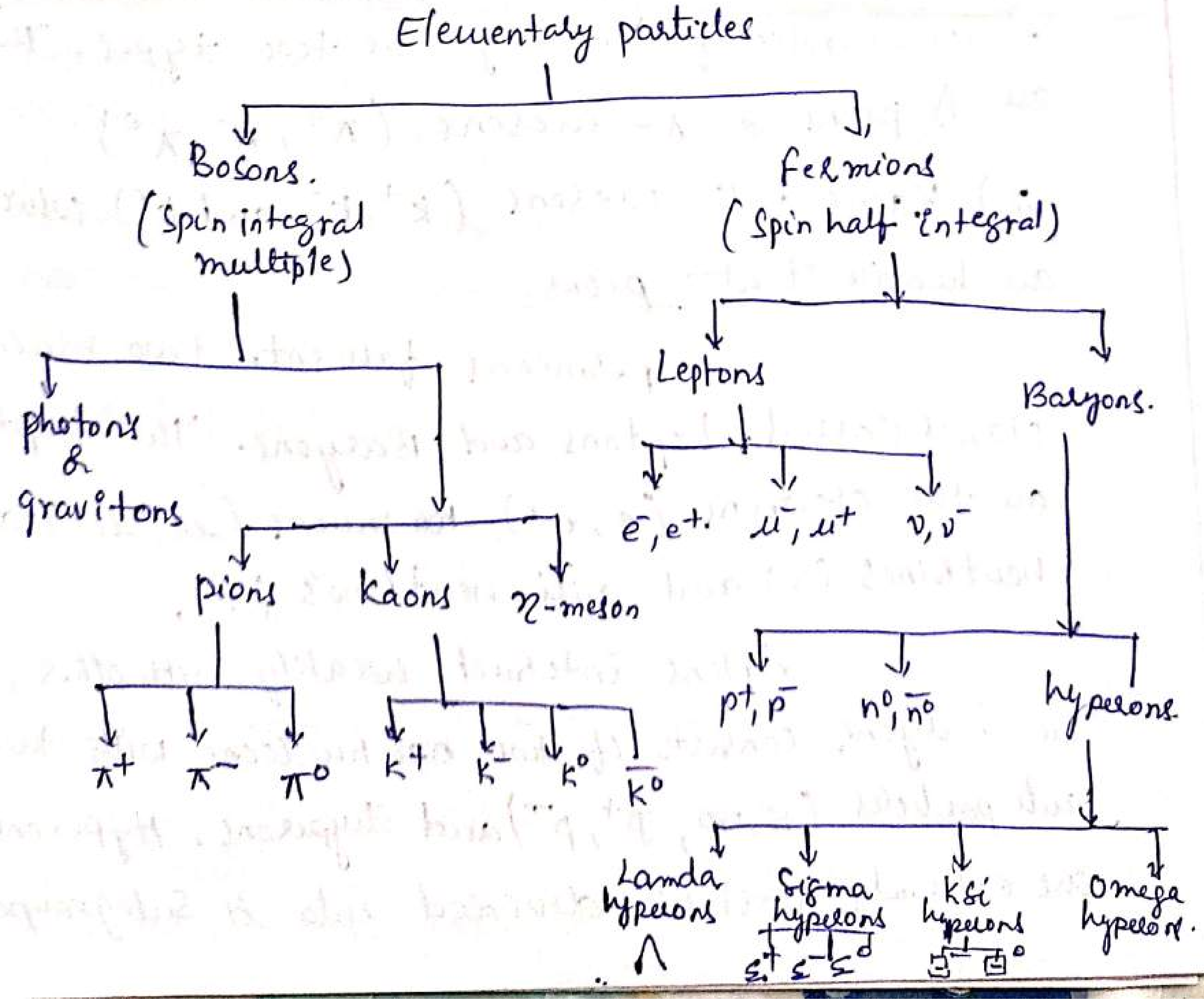
Lepton meson baryons properties

Elementary particles and Interactions :-

→ Discovery and Classification of Elementary particles :-

The elementary particles are nothing but the quanta of a corresponding field. The study of elementary particles is basic to the understanding of radiation phenomena.

Classification of Elementary particles :



(2) The elementary particles are seperated into two general groups. called bosons and fermions. These two groups have different types of spin and diff kind of statistic.

Bosons are particles with intrinsic angular momentum equal to an integral multiple of $\hbar$.

fermions are all those particles in which the spin is half integral. No conservation of law for Bosons in the universe but for fermions is stricctly conserved.

Bosons are simply light photons & x-ray photons. A mass-less boson called a graviton with a probable spin of two units ~~are~~ and bosons are created by electromagnetic field. They are two types. They are 1) pions & π- mesons. (π^+, π^-, π^0)

2) Kaons & K - mesons $(K^+, K^-$, and $K^0)$ which are heavier that pions.

fermions fall into two main classes called Leptons and Baryons. The Leptons are the electrons (e^-, e^+), ~~ka~~ muons (μ^-, μ^+) and neutrinos (ν) and anti neutrino's $(\bar{\nu})$.

Leptons interact weakly with other particles. The baryons consists of two ~~one~~ nucleons with their anti particles $(n^0, \bar{n}^0, p^+, p^-)$ and hyperons. Hyperons are extremly unstable, divided into 4 subgroupers.

Λ^0 particle, the Σ particles ($\Sigma^+, \Sigma^-, \Sigma^0$) the Ξ particles [Ξ^-, Ξ^0] and Ω^- particles. (3)

→ **Types of Interactions [Strong, Electromagnetic Weak Interactions]:-**

The interactions among elementary particles can be classified into following four types.

1) The Gravitational interactions
2) Electromagnetic Interactions
3) Strong Interactions
4) weak Interactions.

Electromagnetic Interactions:-

The strength of the electromagnetic interaction is given by the dimensionless fine structure constant α $\left(= e^2/\epsilon_0 \hbar c = \frac{1}{137}\right)$ and is due to photon exchanges. The electromagnetic interaction is charge dependent. The capture of photon can effect the production of mesons & hyperons by an electro-magnetic interaction

$$\gamma + p \begin{cases} \nearrow \pi^0 + p \\ \searrow \Lambda^0 + K^+ \end{cases}$$

An example of a radiative capture reaction is

$$\pi^- + p \longrightarrow n + \gamma.$$

The neutral particles such as

The neutral particles such as

$$\pi^0 \longrightarrow \gamma + \gamma , \quad \Sigma^0 \longrightarrow \Lambda^0 + \gamma$$

$$\eta^0 \longrightarrow \pi^+ + \pi^- + \pi^0$$

$$\eta^0 \longrightarrow \gamma + \gamma .$$

decay electromagnetically. The electromagnetic interaction is resolved by introducing as an intermediate step in ~~all~~ the overall reaction. Thus we have

$$\pi^0 \xrightarrow[\text{Strong}]{} \underset{\text{Virtual}}{(N + \bar{N})} \xrightarrow[\text{Electromagnetic}]{} \gamma + \gamma$$

The process of mutual annihilation of particles and anti particles is an example of electromagnetic Interaction.

2) <u>Strong Interaction</u>:-

The strong interaction is independent of electric charge. force is same b/w p-p and n-n. The range is very ~~must~~ much shorter than gravitational and electromagnetic interaction.

Strong interaction energy falls off rapidly when the distance b/w two particles increases. The strength of the nuclear interaction is represented by the magnitude of the dimensional coupling const $g^2/4\pi\hbar c$ (≈ 14). It is about a thousand times the electromagnetic coupling const α.

The strong interaction is a short range force ($\approx 10^{-15}$ m) (5)
Strong interactions b/w elementary particles are responsible for total cross sections as a function of energy.

3) Weak interactions:-

The weak interaction is responsible for the decay of strange and non-strange particles and for non-leptonic decays of strange particles.

The numerical const of weak interaction is obtained from fermi's theory of β-decay. Its value is $g_F = 1.41 \times 10^{-62} \, Jm^2$.

$$\left[g_F^2/(\hbar c)^2\right]\left[m_\pi c/\hbar\right]^4 \approx 5 \times 10^{-14}.$$

The other type of weak interactions which require coupling between other pairs of fermions.

$$\Lambda^0 (S=-1) \longrightarrow \pi^- (S=0) + p (S=0).$$

In weak interactions involving change in strangeness of baryons & mesons, the change in strangeness must be equal to the change in charge.

Properties of leptons, mesons and baryons.

→ Conservation Laws :- Isospin, parity, charge conjugation :—

The behaviour of the elementary particles is restricted by a no of conservation laws.

1) Conservation of linear Momentum.
2) Conservation of angular Momentum

(6)

3) Conservation of energy
4) Conservation of charge
5) Conservation of baryon number
6) " " Lepton "
7) " " Isospin
8) " " Hyper charge.
9) " " of Strangeness.
10) charge conjugation.
11) Space Inversion Invariance (parity):-
12) Time Reversal.

1) Conservation of charge:-

~~The charge is conserved in all processes and no exceptions are known. all elementary charges are 0. & ±1, multiple charges are not found~~

1) Conservation of Isospin:-

According to the idea of isotopic spin, each nuclear particle posseses a certain total isotopic Spin T. This isotopic spin along a certain axis T_3 appears to us a a different charge state of the corresponding particle.

In this case of nucleons $T = 1/2$ and $2T+1 = 2$. T_3 as $+1/2$ and $-1/2$. for $T=1$, $2T+1 = 3$ T_3 are $+1, 0, -1$.

$$Q_p = \tfrac{1}{2} + \tfrac{1}{2} = +1, \quad Q_n = -\tfrac{1}{2} + \tfrac{1}{2} = 0$$

$$Q_{\pi^+} = +1 + 0 = 1, \quad Q_{\pi^0} = 0 + 0 = 0, \quad Q_{\pi^-} = -1 + 0 = -1$$

$$Q = T_3 + B/2.$$

Isospin no is associated with hadrons (Strong interaction) but not with leptons. 7

The isospin component T_3 is conserved in both strong and electromagnetic interactions but not in weak interactions.

Charge Conjugation :-

Charge conjugation is defined as the interchange of particles and anti particles. A unitary operator also known as charge conjugation operator C, satisfies the following relations

$$CQC^{-1} = -Q, \quad CYC^{-1} = Y, \quad CBC^{-1} = -B, \quad Cl_eC^{-1} = -l_e$$
$$Cl_\mu C^{-1} = l_\mu$$

Some elementary particles γ and π^0 mesons and positronium atom $(e^- + e^+)$ are transformed into themselves by charge conjugation. They are their own anti particles. These are known as self conjugate & true neutral particles.

The neutron $(B=1, Y=1)$ and K^0 mesons $(Y=1, B=0)$ are not invariant under C.

Parity :-

The parity principle says that there is a symmetry between the world and its mirror image.

This may be defined as reflection of every point in space through the origin of a co-ordinate system.
$x \rightarrow -x$, $y \rightarrow -y$ and $z \rightarrow -z$.

(8) If a system & process is such that its mirror image is impossible to obtain in nature the system of process is said to violate the law of parity conservation.

Eg:- Human body is example for mirror symmetry.

The system can be classified by the eigen values of the parity operator P. for single particle Schroedinger wave function ψ, the result of the parity operation is

$$P\,\psi(x)\rangle = e^{i\alpha}|\psi(+x)\rangle$$

α is arbitrary real phase. hence can be set equal to zero.

$$P|\psi(x)\rangle = |\psi(-x)\rangle$$

$$P^2|\psi(x)\rangle = |\psi(x)\rangle$$

It shows eigen values of P as +1 & −1.

The parity of the photon depends upon the mode of transition, it is due to the change of the sign of electromagnetic current J under parity operation

K meson and η^0 meson - - −ve parity

Λ^0, Ξ^-, Σ^-, Ω^- hyperons have +ve parity

All anti particles of spin ½ are of opposite parity to the corresponding particle, while the bosons and their antiparticles have the same parity.

Mesons:

These are particles heavier than electrons and lighter than nucleons. Their mass lies between 250 and 1000 times mass of the electrons.

- These particles have strong interaction with nuclear matter.
- They are particles of nuclear field.
- They have zero intrinsic spin,
- They are bosons.

Types of Mesons:

Mesons are divided into three groups: π-Meson (pion), k-meson (kaon), and η- meson.

π-Mesons:

π-Mesons are found to have three states of charge:

a) Positive, π^+,
b) Negative, π^- and
c) Neutral, π^0.

Positive π-meson has mass of 275 times mass of electrons and life of about 10^{-2} sec.

Neutral π-meson has mass of 264 times mass of the electron.

Neutral π^0 mesons are produced in the collision of protons having Kinetic energy $\geq$ 150 MeV. Similarly, in photoproduction of energetic gamma quanta, π^{+-} mesons are produced.

K-Mesons:

K-Mesons are found to be in three charge states:

a) Positive, K^+, b) Negative, K^-, and c) Neutral, K^0

Positive K-meson has mass 966 times of the electronic mass. And that neutral K- mesons have 967 times electronic mass.

Baryons:

These are heavy particles. Their intrinsic spin is odd-half integer spin. Baryons are further grouped into two classes:

a) nucleon, and b) hyperons.

Nucleons are constituent particles of the nucleus of an atom. They are protons and neutrons. Protons and neutrons have nearly equal mass. But protons are positively charged and neutrons are neutral.

Hyperons:

Hyperons are the particles heavier than nucleons. They are characterized by their lifetime, which has the order of 10^{-10} sec. The hyperons have strange behaviour that they decay faster than are formed.

Types of hyperons:

There are four types of hyperons:

Λ-hyperon, Σ-hyperons, Ξ-hyperons and Ω-hyperons.

Λ-hyperons are neutral particles and so are represented as Λ^0

Σ-hyperons exist in three charge states

a) Σ^+, b) Σ^-, and Σ^0,

Ξ-hyperon in two charge states:

a) Positive Ξ^+, and (b) Negative Ξ^-

Ω-hyperon exits in only one charge state Ω^-.

Antiparticles:

Antiparticle of a particle is the particle having the same mass but of opposite charge. When the particle and its antiparticle come in contact they annihilate each other with the emission of photons. All the particles have their antiparticle.

Positron is the first discovered antiparticle of electron. The antiproton is discovered by Segre, Chamberlain and other in the collision of high energy protons.

The antiparticle of neutron, the antineutron, was discovered in 1956 by Cork, Lamberton and Wengel.

Neutrino and antineutrino are distinguished by their spins. Neutrino spins counterclockwise while antineutrino clockwise.

3. What are Leptons? Give the properties of leptons.

Ans. Leptons are a class of elementary particles that are fundamental to the Standard Model of particle physics. They do not participate in strong interactions, but they interact through the weak force, electromagnetic force (if charged), and gravitational force. Below are the key properties of leptons:

1. Types of Leptons:

Leptons are divided into three generations, with each generation containing a charged lepton and a corresponding neutrino. These are:

a) First Generation:

i) Electron (e): A charged lepton with a negative charge.

ii) Electron neutrino (ν_e): A neutral lepton with extremely small mass, associated with the electron.

b) Second Generation:

i) Muon (μ): A charged lepton similar to the electron but much heavier.

ii) Muon neutrino (ν_π): A neutral lepton, associated with the muon.

c) Third Generation:

i) Tau (τ): A charged lepton heavier than the muon.

ii) Tau neutrino (ν_τ): A neutral lepton, associated with the tau.

2. Characteristics:

a) Mass: Leptons have different masses depending on their generation. The electron is the lightest, while the tau is the heaviest. The neutrinos, on the other hand, have extremely small masses that are not yet precisely determined but are known to be much smaller than the masses of the charged leptons.

b) Charge: All leptons except for neutrinos have electric charge. The charged leptons (electron, muon, tau) each have a negative electric charge, while the neutrinos are electrically neutral.

c) Spin: Leptons are fermions, meaning they have a half-integer spin value (½).

d) Stability:

i) The electron is stable and exists freely in nature.

ii) The muon and tau are unstable and decay into other particles after a very short time (muons decay in around 2.2×10^{-6} seconds, while tau leptons decay in around 2.9×10^{-13} seconds).

e) Neutrinos:

i) Neutrinos (electron neutrino, muon neutrino, tau neutrino) are neutral, lightweight particles that interact very weakly with other matter. As a result, they pass through ordinary matter with very little interaction, making them extremely difficult to detect.

ii) Neutrinos are produced in weak interactions, such as during nuclear decay or in particle collisions.

3. Interactions:

Leptons can participate in different types of interactions:

a) Electromagnetic Interaction:

i) The charged leptons (electron, muon, tau) interact via the electromagnetic force, which is mediated by the photon. This means they can be affected by electric and magnetic fields.

ii) The neutrinos do not interact electromagnetically as they are electrically neutral.

b) Weak Interaction:

i) All leptons (charged and neutral) interact via the weak force, mediated by the W and Z bosons. This interaction is responsible for processes like beta decay.

ii) Neutrinos are involved in weak interactions, which are central to their detection.

c) Gravitational Interaction:

Like all matter, leptons are affected by gravity. However, their interaction with gravity is extremely weak compared to the other forces.

4. Lepton Number Conservation:

a) Lepton number is a quantum number that is conserved in most particle interactions. In other words, the total number of leptons minus the total number of antileptons remains constant in an interaction.

b) There are three different lepton numbers corresponding to each generation of leptons:

i) **Electron lepton number (L_e)** for electron and electron neutrino.

ii) **Muon lepton number (L_π)** for muon and muon neutrino.

iii) **Tau lepton number (L_τ)** for tau and tau neutrino.

c) In reactions, if a particle is created or destroyed, its corresponding lepton number must be balanced by the corresponding lepton number of other particles.

5. Neutrino Oscillations:

Neutrinos are known to undergo a phenomenon called neutrino oscillation, where neutrinos can change from one type (flavor) to another as they travel. This implies that neutrinos have a tiny but non-zero mass, which was experimentally confirmed in the late 1990s through experiments such as the Super-Kamiokande and SNO experiments.

4. What are Mesons? Give the properties of Mesons.

Ans. Mesons are composite particles made up of a quark-antiquark pair and belong to the family of hadronic particles (which also includes baryons). Unlike leptons, mesons interact via the strong force, which binds their quark constituents together. Mesons are bosons, meaning they have integer spin.

Key Characteristics of Mesons:

1. Quark Composition: Mesons consist of one quark and one antiquark. These quark pairs can be made from any combination of the six types (flavors) of quarks: up (u), down (d), charm (c), strange(s), top (t), and bottom (b). However, the top and bottom quarks are much too massive to form stable mesons.

2. Spin: Mesons always have an integer spin, which means they are bosons. The most common mesons have a spin of 0 or 1.

3. Mass: Mesons typically have masses ranging between the mass of a proton (about 938 MeV/c^2) and the tau lepton (about 1777 MeV/c^2). The masses of mesons depend on the types of quarks involved in their composition.

4. Lifetime: Mesons are unstable and decay into lighter particles over a short time, ranging from very tiny fractions of a second to a few microseconds, depending on the specific meson.

5. Role in Fundamental Interactions: Mesons are responsible for mediating the strong interaction between baryons (such as protons and neutrons) in the atomic nucleus. This interaction is crucial for holding atomic nuclei together.

Types of Mesons:

Mesons are divided into two main categories based on their charge and spin:

I. Pions (π-mesons): Pions are the lightest and most well-known mesons. There are three types of pions:

a) **Charged Pions:**

i) π^+: Composed of an up quark and an anti-down quark.

ii) π^-: Composed of a down quark and an anti-up quark.

b) **Neutral Pion:**

i) π^0: Composed of a combination of up and anti-up quarks or down and anti-down quarks in a quantum superposition.

Properties of Pions:

1. **Mass:** π^+, π^-, and π^0 have masses around 140 MeV/c^2.

2. **Spin:** 0 (pions are bosons).
3. **Charge:**
   a) π^+ is positively charged.
   b) π^- is negatively charged.
   c) π^0 is neutral.
4. **Lifetime:** Pions are unstable. The charged pions decay into muons and neutrinos, and neutral pions decay primarily into photons.

Role:

Pions mediate the strong nuclear force that binds protons and neutrons in the nucleus. They were crucial in confirming the quark model and the concept of quantum chromodynamics (QCD).

II. Kaons (K-mesons)

Kaons are mesons that contain a strange quark. There are several types of kaons:

a) **Charged Kaons:**

i) K^+: Composed of an up quark and an anti-strange quark.

ii) K^-: Composed of a strange quark and an anti-up quark.

b) **Neutral Kaons:**

i) K^0: Composed of a down quark and an anti-strange quark.

ii) $\bar{K}^0$: Composed of a strange quark and an anti-down quark.

Properties of Kaons:

1. **Mass:** Kaons have masses between 494 MeV/c^2 (for K^0) and 498 MeV/c^2 (for K^+).
2. **Spin:** 0 (kaons are bosons).
3. **Charge:**
   i) K^+ is positively charged.
   ii) K^- is negatively charged.
   iii) K^0 is neutral.
4. **Lifetime:**

Kaons are relatively long-lived, with lifetimes ranging from 10^{-10} to 10^{-8} seconds. They decay via weak interactions into various lighter particles, such as pions, neutrinos, and photons.

Role:

Kaons are important in understanding the violation of CP symmetry (the combined symmetry of charge conjugation and parity), which plays a key role in the study of CP violation and the imbalance between matter and antimatter in the universe.

III. Charm Mesons:

Charm mesons contain charm quarks. Some examples are:

1. D^+: Composed of a charm quark and an anti-down quark.
2. D^0: Composed of a charm quark and an anti-up quark.
3. D_s^+: Composed of a charm quark and an anti-strange quark.

Properties of Charm Mesons:

1. **Mass:** Around 1.86 GeV/c^2 for D^+ and 1.97 GeV/c^2 for D^+_s.
2. **Spin:** 0 (bosons).
3. **Lifetime:**

Charm mesons are relatively stable compared to pions and kaons but still decay quickly via the weak force, with lifetimes on the order of 10^{-13} seconds.

Role:

Charm mesons are essential for the study of the strong interaction and the charm quark's role in particle physics, especially in the context of the Standard Model and quantum chromodynamics (QCD).

IV. Bottom Mesons:

Bottom mesons contain bottom quarks. Examples include:

1. B^+: Composed of a bottom quark and an anti-up quark.
2. B^0: Composed of a bottom quark and an anti-down quark.
3. B_s^0: Composed of a bottom quark and an anti-strange quark.

Properties of Bottom Mesons:

1. **Mass:** Around 5.3 GeV/c^2 for B^+ and 5.4 GeV/c^2 for B^0_s.
2. **Spin:** 0 (bosons).
3. **Lifetime:** Bottom mesons have relatively long lifetimes, on the order of 10^{-12} seconds, and decay via the weak force.

Role: Bottom mesons are used to study CP violation and B physics, which has provided significant insights into the nature of the weak force and the behavior of heavy quarks.

5. What are Baryons? Give the properties of Baryons.

Ans. Baryons are composite particles made up of three quarks bound together by the strong force (mediated by gluons). Baryons belong to the family of hadronic particles, along with mesons, and they are fermions (particles with half-integer spin). Baryons include familiar particles like protons and neutrons, which make up the atomic nucleus.

Key Characteristics of Baryons:

1. **Quark Composition:**
   a) Baryons consist of **three quarks.** These quarks can be any combination of the six available quark flavors: up (u), down (d), charm (c), strange (s), top (t), and bottom (b).
   b) The most common baryons in nature are composed of up and down quarks.
2. **Spin:** Baryons are fermions with half-integer spin (½).
3. **Mass:**

Baryons typically have masses ranging from around 938 MeV/c^2 (for the proton) to several GeV/c^2 (for heavier baryons containing charm or bottom quarks).

4. **Lifetime:**
   a) Baryons can either be stable (like the proton) or unstable (like certain heavier baryons that decay via weak interactions).
   b) The proton is stable in most scenarios, while other baryons with heavy quarks (like bottom baryons) have short lifetimes.
5. **Charge:**

Baryons can be electrically charged or neutral, depending on the quark combination:

i) Positive charge: Composed of two up quarks and one down quark (e.g., proton).

ii) Negative charge: Composed of two down quarks and one up quark (e.g., neutron).

6. **Interaction:**

Baryons interact via the strong force (the force holding the quarks together) and also interact via the weak force and electromagnetic force (if they are charged).

Examples of Baryons:

1. **Proton (p):**

a) Quark Composition: uud (two up quarks, one down quark).

b) Charge: +1 (positively charged).

c) Mass: 938.27 MeV/c^2.

d) Spin: ½ (fermion).

e) Lifetime: Stable (in the absence of extreme conditions like those found in particle accelerators).

f) Role: Protons are the building blocks of atomic nuclei. They are involved in the strong interaction and electromagnetic interaction.

2. Neutron (n):

a) **Quark Composition:** udd (one up quark, two down quarks).

b) **Charge:** 0 (neutral).

c) **Mass:** 939.57 MeV/c^2 (slightly heavier than the proton).

d) **Spin:** ½ (fermion).

e) **Lifetime:** Unstable outside the nucleus (decays into a proton, electron, and neutrino in about 10 minutes).

f) **Role:** Neutrons, along with protons, form the nucleus of atoms. They contribute to the strong nuclear force that binds nucleons together in the nucleus.

3. Lambda Baryons (Λ, Σ, Ξ):

These are baryons containing at least one strange quark. They are examples of hyperons, which are baryons that contain strange, charm, or bottom quarks.

a) Lambda Baryons (Λ):

i) **Quark Composition:** uss (one up quark and two strange quarks).

ii) **Charge:** 0.

iii) **Mass:** Around 1115 MeV/c^2.

iv) **Lifetime:** About 2.6×10^{-10} seconds.

v) **Spin:** ½ (fermion).

b) Sigma Baryons (Σ):

i) **Quark Composition:** Can contain combinations of up, down, and strange quarks (e.g., uus, uds, dss).

ii) **Charge:** +1, 0, or –1 depending on the specific quark content.

iii) **Mass:** Around 1192 MeV/c^2.

iv) **Lifetime:** Short-lived, decaying via weak interactions.

c) Xi Baryons (Ξ):

i) **Quark Composition:** Contains two strange quarks (e.g., uss, dss).

ii) **Charge:** – 1 or 0.

iii) **Mass:** Around 1315 MeV/c^2.

iv) **Lifetime:** Short-lived (similar to sigma baryons).

4. Heavy Baryons (Charm and Bottom):

These baryons contain charm (c) or bottom (b) quarks, which are much heavier than the up and down quarks.

i) **Charm Baryons (e.g., Λ_c):**Contain at least one charm quark.

ii) **Bottom Baryons (e.g., Λ_b):** Contain at least one bottom quark.

These baryons are unstable and decay via the weak interaction into lighter particles, often through a series of intermediate decays.

6. Explain about fundamental particle interactions.

Ans. In particle physics, the fundamental forces that govern the behaviour of subatomic particles are categorized into four types: gravitational, electromagnetic, strong, and weak interactions.

1. Strong Interaction (or Strong Nuclear Force): The strong interaction is the force that holds atomic nuclei together. It is the most powerful of the four fundamental forces but acts over very short distances of about 10^{-15}meters.

Strong interaction force is mediated by gluons, which are exchanged between quarks inside protons and neutrons. In nuclei, the strong interaction is responsible for holding protons and neutrons together, and it also binds quarks together to form protons and neutrons.

Characteristics:

1. The strong interaction is attractive at short distances (around the size of the nucleus). The strong force is short-range, it becomes negligible at distances larger than the size of the nucleus.
2. It is charge-independent, it does not depend on the electric charge of the particles involved. It is same for (p–n), (p–p), (p–n) interactions.
3. It is asymmetric. It is much stronger between quarks than between nucleons (protons and neutrons).
4. It is spin-dependent and iso-spin dependent, and also depends on the velocity.
5. It is independent of relative orientation of the nucleons.
6. Colour charge: Unlike electric charge in electromagnetism, the strong force operates between particles that carry a property called "colour charge." This is a feature of quarks and gluons.
7. It doesn't obey inverse square law. Its characteristic time is of the order of 10^{-23} sec.
8. This interaction is described by Quantum Chromo dynamics (QCD), which involves the exchange of gluons between quarks. Quarks come in different "colours," and gluons are exchanged to keep quarks confined inside hadrons (protons, neutrons, etc.).

Ex: The force between two protons in the nucleus is due to the strong interaction. Even though both having positive electric charge and thus repelling each other electromagnetically, they are held together by the strong force.

2. Electromagnetic Interaction: The electromagnetic interaction is the force that acts between charged particles. It is responsible for the behaviour of electrically charged particles and governs phenomena like electricity, magnetism, and light.

The electromagnetic interaction force is mediated by photons, which are the quanta of electromagnetic fields.

Characteristics:

1. **Long-range:** The electromagnetic force acts over long distances, effectively infinite in theory, though it weakens with distance according to Coulomb's law.
2. **Charge-dependent:** The force acts between particles with electric charge. Like charges repel, and opposite charges attract.
3. The electromagnetic force is the same in a vacuum as it is in other media (though materials can influence how the force behaves).
4. It can be attractive between opposite charges or repulsive between like charges.
5. It governs interactions between charged particles, such as electrons and protons. It is responsible for the behaviour of light and electromagnetic radiation, as well as chemical bonds and electricity.
6. This interaction is described by Quantum Electrodynamics (QED), which is the quantum field theory of electromagnetism.

   The force is mediated by photons, which are mass less and travel at the speed of light.

Ex: The force between two electrons is purely electromagnetic. Since electrons have the same negative charge, they repel each other due to the electromagnetic force.

3. Weak Interaction (Weak Nuclear Force): The weak interaction is responsible for certain types of particle decay (such as beta decay) and the interactions between elementary particles that change flavour (such as the conversion of a neutron into a proton).

The weak interaction force is mediated by the W and Z bosons, which are massive particles. The W bosons come in two types: (W^+) and (W^-), and the Z boson is neutral.

Characteristics:

1. The weak interaction force operates only over very short distances of the order of (10^{-18}) meters, much smaller than the size of an atomic nucleus.
2. It is responsible for the transmutation of one type of quark or lepton into another (for example, a neutron can decay into a proton, emitting a beta particle and an antineutrino).
3. In this conservation of parity is violated. (i.e., the weak force does not behave the same when spatial coordinates are inverted), a unique property in comparison to the strong and electromagnetic forces, which do conserve parity.
4. The weak interaction force is described by the Electroweak Theory, which combine the electromagnetic and weak forces. This theory is part of the Standard Model of particle physics.
5. The weak force can change the type (flavour) of quarks, which is important for processes like beta decay, where a down quark is converted into an up quark.

Ex: The weak interaction is crucial for nuclear processes, such as beta decay in radioactive materials. In this process, a neutron can decay into a proton, electron, and an electron antineutrino.

$n \rightarrow p^+ + e^- + \bar{\nu}$

These interactions are fundamental to the structure and behaviour of matter, from the creation of atoms to the reactions that power stars.

7. Write a note on Conservation Laws in Particle Physics.

Ans. Conservation laws are vital principles in particle physics, certifies the consistency and predictability of particle interactions. These laws provide the foundation for understanding particle decays, reactions, and the behavior of fundamental forces in nature.

1. Conservation of Energy:

a) Statement: The total energy in a closed system remains constant over time.

b) Application: Energy can be converted from one form to another (e.g., from mass to kinetic energy or vice versa) but cannot be created or destroyed.

c) Formula: $E = mc^2$ (Einstein's famous equation), where E is energy, m is mass, and c is the speed of light.

SHORT ANSWER QUESTIONS

9. Write a short note on Lepton properties.

Ans.

Property	Electron	Muon	Tau	Electron Neutrino	Muon Neutrino	Tau Neutrino
Charge	– 1	– 1	– 1	0	0	0
Mass	9.11×10^{-31} kg	1.88×10^{-28} kg	3.18×10^{-27} kg	Extremely small	Extremely small	Extremely small
Spin	½	½	½	½	½	½
Life-time	Stable	Unstable (2.2×10^{-6} seconds)	Unstable (2.9×10^{-13} seconds)	Stable (no decay observed)	Stable (no decay observed)	Stable (no decay observed)
Interactions	Electromagnetic weak, gravitation	Electromagnetic weak, gravitation	Electromagnetic weak, gravitation	Weak, gravitational	Weak, gravitational	Weak, gravitational

Leptons are fundamental particles that play an essential role in the universe, from the formation of atoms to weak nuclear processes, and understanding them is crucial for our knowledge of particle physics and the universe at large.

10. Write a short note on properties of mesons.

Ans. Mesons are composite particles made of quark-antiquark pairs and are bosons with integer spin. They play a crucial role in mediating the strong interaction between baryons (protons and neutrons).

Pions are the lightest mesons and play a key role in nuclear interactions. Kaons, charm mesons, and bottom mesons are heavier mesons and are vital for studying the strong force, CP violation, and the behavior of different quarks. Understanding mesons is key to the study of particle physics and the fundamental forces of nature.

Summary of Meson Properties:

Property	Pions (π)	Kaons (K)	Charm Mesons (D)	Bottom Mesons (B)
Quark Composition	Up and down quarks (π^+, π^-, π^0)	Strange and non-strange quarks (K^+, K^-, K^0)	Charm quark and anti-quark	Bottom quark and anti-quark
Mass	140 MeV/c^2 (π^+, π^-)	494 – 498 MeV/c^2	1.86–1.97 GeV/c^2	5.3–5.4 GeV/c^2
Spin	0	0	0	0
Charge	± 1 (π^+, π^-), 0 (π^0)	± 1 (K^+, K^-), 0 (K^0)	± 1 (D^+, D^0)	± 1 (B^+, B^0)
Lifetime	10^{-8} to 10^{-10} seconds	10^{-10} to 10^{-8} seconds	10^{-13} seconds	10^{-12} seconds
Interaction	Strong force mediation	Strong force mediation	Strong force mediation	Strong force mediation

11. Write a short note on characteristics of Baryons.

Ans. Baryons are fundamental particles made of three quarks. They include the proton and neutron as well as more exotic particles like hyperons (containing strange, charm, or bottom quarks). Baryons interact via the strong force, and while many are unstable, protons are stable and crucial for matter as we know it.

Property	Proton (p)	Neutron (n)	Lambda Baryons (Λ)	Sigma Baryons (Σ)	Xi Baryons (Ξ)	Heavy Baryons (Λ_c, Λ_b)
Quark Composition	uud	udd	uss	uus, uds, dss	uss, dss	ucq, bcq
Charge	+1	0	0	+1, 0, –1	–1, 0	+1, 0
Mass	938 MeV/c^2	939 MeV/c^2	1115 MeV/c^2	1192 MeV/c^2	1315 MeV/c^2	2.4 GeV/c^2 (approx.)
Spin	½	½	½	½	½	½

Lifetime	Stable	Unstable (10 min.)	Short-lived ($\sim 10^{-10}$ sec.)	Short-lived (about 10^{-10} sec.)	Short-lived (about 10^{-10} sec.)	Short-lived (about 10^{-12} sec.)
Interaction	Strong, weak, electromagnetic (if charged)	Strong, weak, electromagnetic (if charged)	Strong, weak	Strong, weak	Strong, weak	Strong, weak

12. Write the differences between a hadron and a lepton.

Ans.

Feature	Hadrons	Leptons
Composition	Made of quarks (composite)	Elementary particles (not composite)
Interactions	Strong, weak, electromagnetic, gravity	Electromagnetic, weak, gravity
Mass	Generally heavy	Lighter (e.g., electron is much lighter than a proton)
Size	Finite size (due to quark structure)	Point-like (no size)
Stability	Some stable (e.g., proton), others unstable (e.g., neutron)	Stable (electron) or unstable (muon, tau)
Examples	Proton, neutron, pion, kaon	Electron, muon, tau, neutrinos
Quantum Numbers	Baryon number, charge, isospin, strangeness	Lepton number, charge, spin

13. Mention the properties of Baryons.

Ans. Baryons are subatomic particles with several properties, including:

1. Composition: Baryons are made up of three quarks. The term "baryon" usually refers to triquarks, which are baryons made of three quarks.

2. Baryon number: Baryons have a baryon number of 1, while their antiparticles, called antibaryons, have a baryon number of −1.

3. Spin: The three quarks that make up a baryon can only produce half-integer values.

4. Colorless: The combination of three quarks of different colors (red, green, blue) gives white.

5. Strong interactions: Baryons are responsible for strong interactions.

6. Fermions: Baryons are fermions, which means they are described by Fermi-Dirac statistics and obey the Pauli exclusion principle.

7. Hadrons: Baryons are hadrons, which are composite particles made up of quarks.

Examples:

Protons and neutrons are examples of baryons. Other baryons include the lambda, sigma, xi, and omega particles.

14. Compare strong, electromagnetic and weak interactions.

Ans.

Interaction	Mediating Particle	Strength	Range	Type of Particles Affected
Strong Force	Gluon	Strongest	Very short (10^{-15} meters)	Quarks
Electro-magnetic Force	Photon	Moderate	Infinite (decreases with distance)	Charged particles
Weak Force	W and Z bosons	Weak	Very short (10^{-18} meters)	Quarks & leptons

15. Write a short note on Isospin.

Ans. Isospin or **isotopic spin quantum number** is a quantum number related to the **strong nuclear force.** It was introduced by **Heisenberg** to explain the similarities between protons and neutrons under the strong force. Protons and neutrons have different electric charges, but they behave similarly in terms of their strong interaction because they are both nucleons. Isospin was thus developed as a way to describe these particles in a manner analogous to spin.

In isospin, the proton and neutron are treated as two different states of a single particle called the nucleon. The conservation of isospin is crucial for understanding the interactions of hadrons (particles like protons, neutrons, mesons, etc.) under the strong force.

Isospin Quantum Number (I):

Isospin is described by the quantum number I, similar to how regular spin is described by the quantum number S. The isospin quantum number I characterizes the symmetry of a particle under the strong force. The z-component of the isospin, I_z, represents the difference in the number of up and down quarks (or antiquarks) within a particle, and is related to the electric charge Q of the particle.

1. For a system with isospin I, the value of I_z can range from $-I$ to $+I$ in integer steps, i.e., $I_z = I, I-1, ..., -I$.
2. The electric charge Q is related to I_z and the isospin I by the relation:

$$Q = I_z + \frac{1}{2}(N - A)$$

where N is the number of neutrons and Z is the number of protons in the nucleus (this formula is used in nuclei, but the concept is the same for individual particles).

UNIT IV

NUCLEAR DECAYS AND NUCLEAR ACCELERATORS

ESSAY QUESTIONS

1. Explain Gamow's Theory of α–Decay with necessary theory.

Ans. The α-particle can come out from the nucleus of U^{238} if its energy is more than 35.6 MeV. But, experimentally it is found that the kinetic energy of α-particle from U^{235} is about 4 MeV. So, according to classical theory, an α-particle cannot escape from the nucleus. Hence classical theory fails to explain α-decay. Using quantum mechanics, Gamow had shown that there is a finite probability for an α-particle possessing an energy less than the potential barrier to penetrate the potential barrier and escape from the nucleus. Gamow made the following assumptions:

1. The α-particle is pre-formed inside the parent nucleus.
2. The α-particle is in constant motion and bounces back and forth from the barrier walls. In each collision with the wall of potential barrier, there is a definite probability of leakage through the barrier.

Let P be the probability of transmission of α-particle in each collision. Let ν be the frequency with which the α-particle collides with the walls in order to escape from the nucleus. Now, the decay probability per unit time (disintegration constant λ) is given by

$$\lambda = \nu P \qquad \text{..... (1)}$$

If there by only one α-particle in the nucleus which moves back and forth along the nuclear diameter, then $\nu = \frac{\upsilon}{2r_0}$ .....(2)

where υ is the velocity of α-particle and r_0 be the nuclear radius.

$$\therefore \lambda = \left(\frac{\upsilon}{2r_0}\right)P \qquad \text{..... (3)}$$

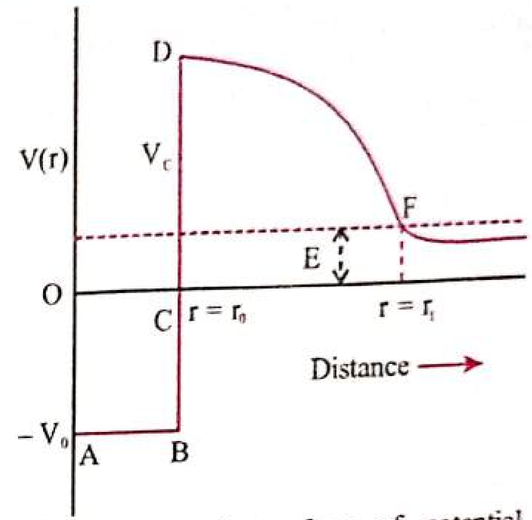


Fig. represents approximate form of potential energy of α-particle as a function of its distance from the nucleus. The potential that exists around the nucleus is known as potential well. In the figure, $OA = -V_0$, is potential energy of α-particle when it is inside the nucleus, CD is equal to V_C, the electrostatic potential energy of α-particle when it is just outside the nucleus, it is unable to enter the nucleus. The reason being that the particle gets repelled back away from the nucleus. On the otherhand, if the particle is inside the nucleus, it is also unable to escape from the nucleus, because it is pulled back into the nucleus. But quantum mechanically, a moving particle is treated as a wave and there is a definite probability to leak through the barrier. This is called Tunnel effect. The α-particle has a finite probability of being able to cross the potential hill.

Using WKB approximation, the probability P can be calculated by the following formula,

$$\log_e P = -\frac{2}{\hbar}\int_{r_0}^{r_1} \sqrt{\{2m(V(r)-E\}}dr \qquad \text{..... (4)}$$

where m is the mass of the particle and $\hbar = h/2\pi$. Let V(r) be the electrostatic potential energy of α-particle at a distance r from the nucleus of charge Ze. The charge on α-particle is 2 e.

$$\text{Hence } V(r) = \frac{2Ze^2}{4\pi\varepsilon_0 r} \qquad \text{......(5)}$$

$$\therefore \log_e P = -\frac{2}{\hbar}\int_{r_0}^{r_1}\sqrt{\left\{2m\left(\frac{2Ze^2}{4\pi\varepsilon_0 r}\right)-E\right\}}\,dr \quad \ldots\ldots(6)$$

At $r = r_1$, $E = \frac{2Ze^2}{4\pi\varepsilon_0 r_1}$

$$\therefore \log_e P = -\frac{2}{\hbar}\int_{r_0}^{r_1}\sqrt{\left\{2m\left(\frac{Er_1}{r}-E\right)\right\}}\,dr$$

$$\Rightarrow \log_e P = -\frac{2}{\hbar}(2mE)^{1/2}\int_{r_0}^{r_1}\left(\frac{r_1}{r}-1\right)^{1/2}dr \ldots\ldots(7)$$

Consider the integral, $\int_{r_0}^{r_1}\left(\frac{r_1}{r}-1\right)^{1/2}dr$

Let us substitute $r = r_1\cos^2\theta$

$\therefore dr = -2r_1 \cos\theta \sin\theta\, d\theta$

Let $r_0 = r_1\cos^2\theta_0$

$$\therefore \int_{r_0}^{r_1}\left(\frac{r_1}{r}-1\right)dr = \int_{\theta_0}^{\theta}\left(\frac{r_1}{r_1\cos^2\theta}-1\right)^{1/2}(-2r_1\cos\theta\sin\theta\, d\theta)$$

$$= -2r_1\int_{\theta_0}^{\theta}\frac{(1-\cos^2\theta)^{1/2}}{\cos\theta}\cos\theta\sin\theta\, d\theta$$

$$= -2r_1\int_{\theta_0}^{\theta}(1-\cos^2\theta)^{1/2}\sin\theta\, d\theta$$

$$= -2r_1\int_{\theta_0}^{\theta}\sin^2\theta\, d\theta$$

$$= -2r_1\int_{\theta_0}^{\theta}\frac{1}{2}(1-\cos 2\theta)\, d\theta$$

$$= -r_1\left[\theta - \frac{\sin 2\theta}{2}\right]_{\theta_0}^{\theta}$$

$$= -r_1\left[-\theta_0 + \frac{1}{2}\sin 2\theta_0\right]$$

$$= r_1[\theta_0 - \sin\theta_0\cos\theta_0]$$

As $\cos^2\theta_0 = \frac{r_0}{r_1}$

$$\Rightarrow \theta_0 = \cos^{-1}\sqrt{\left(\frac{r_0}{r_1}\right)} \text{ and } \sin\theta_0 = \sqrt{1-\cos^2\theta_0} = \sqrt{\left(1-\frac{r_0}{r_1}\right)}$$

$$\therefore \int_{r_0}^{r_1}\left(\frac{r_1}{r}-1\right)^{1/2}dr = r_1\left[\cos^{-1}\sqrt{\left(\frac{r_0}{r_1}\right)} - \sqrt{\left(1-\frac{r_0}{r_1}\right)}\sqrt{\left(\frac{r_0}{r_1}\right)}\right] \quad \ldots..(8)$$

From equations (7) and (8), we get

$$\log_e P = -\frac{2}{\hbar}(2mE)^{1/2}r_1\left[\cos^{-1}\sqrt{\left(\frac{r_0}{r_1}\right)} - \sqrt{\left(1-\frac{r_0}{r_1}\right)}\sqrt{\left(\frac{r_0}{r_1}\right)}\right] \quad \ldots..(9)$$

As the potential barrier is relatively wide ($r > r_0$), hence

$$\cos^{-1}\left(\frac{r_0}{r_1}\right)^{1/2} \approx \frac{\pi}{2} - \left(\frac{r_0}{r_1}\right)^{1/2} \text{ and } \left(1-\frac{r_0}{r_1}\right)^{1/2} \approx 1$$

Substituting these values in eq. (9), we get

$$\log_e P = -\frac{2}{\hbar}(2mE)^{1/2}r_1\left[\frac{\pi}{2} - \left(\frac{r_0}{r_1}\right)^{1/2} - \left(\frac{r_0}{r_1}\right)^{1/2}\right]$$

$$= -\frac{2}{\hbar}(2mE)^{1/2}r_1\left[\frac{\pi}{2} - 2\left(\frac{r_0}{r_1}\right)^{1/2}\right] \quad \text{Here } r_1 = \frac{2Ze^2}{4\pi\varepsilon_0 E}$$

$$\therefore \log_e P = -\frac{2}{\hbar}(2mE)^{1/2}\frac{2Ze^2}{4\pi\varepsilon_0 E}\left[\frac{4}{2} - 2x\left(\frac{\pi\varepsilon_0 E r_0}{2Ze^2}\right)^{1/2}\right]$$

$$\log_e P = \frac{4e}{\hbar}\left(\frac{m}{\pi\varepsilon_0}\right)^{1/2}Z^{1/2}r_0^{1/2} - \frac{e^2}{\hbar\varepsilon_0}\left(\frac{m}{2}\right)^{1/2}ZE^{-1/2} \quad \ldots..(10)$$

Substituting the values of various constants in eq. (10), we have

$$\log_e P = 2.97\, Z^{1/2} r_0^{1/2} - 3.95\, ZE^{-1/2} \quad \ldots..(11)$$

Here E = Kinetic Energy in MeV, r_0 = nuclear radius in fermi and Z = atomic number of nucleus – alpha particle

The decay constant is given by $\lambda = vp = \left(\frac{\upsilon}{2r_0}\right)P$

$$\log_e \lambda = \log_e\left(\frac{\upsilon}{2r_0}\right) + \log_e P$$

$$= \log_e\left(\frac{\upsilon}{2r_0}\right) + 2.97\, Z^{1/2} r_0^{1/2} - 3.95\, ZE^{-1/2} \quad \ldots..(12)$$

Eq. (12) represents the α-decay.

When base of $\log_e$ is changed to base_{10}, we have

$$\log_{10}\lambda = \log_{10}\left(\frac{\upsilon}{2r_0}\right) + 1.29\, Z^{1/2} r_0^{1/2} - 1.72\, ZE^{-1/2} \quad \ldots\ldots(13)$$

Expression (13) is regarded as theoretical form of Geiger-Nuttal Law. Gamow's theory explains the essential features of α-decay but the assumption of pre-existence of an α-particle in the parent nucleus was found to be incorrect.

2. Explain Fermi's theory of Beta- decay.

Ans. Fermi's theory of beta decay is a quantum mechanical description of the process by which a neutron decays into a proton, emitting an electron (beta particle) and an antineutrino.

The Process of Beta Decay:

In beta decay, a neutron (composed of two down quarks and one up quark) decays into a proton (composed of two up quarks and one down quark), emitting a beta particle (electron) and an antineutrino. The process can be described as:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

where:

n is the neutron,

p is the proton,

e^- is the emitted electron (beta particle),

$\bar{\nu}_e$ is the antineutrino.

Fermi postulated that the process of beta decay is mediated by the **weak force**, which is one of the four fundamental forces in nature. The weak force is responsible for interactions that change the flavour of quarks (such as the transformation of a down quark into an up quark in the neutron decay).

Fermi described the interaction responsible for beta decay using a four-fermion interaction. In this model, the decay occurs through the interaction of four particles (the neutron, proton, electron, and antineutrino) in a single point, which is an essential feature of the weak interaction at low energies. The interaction is described by the following Hamiltonian:

$$H_{int} = \frac{G_F}{\sqrt{2}}\left(\overline{\psi}_p \gamma_\mu (1-\gamma_5)\psi_n\right)\left(\overline{\psi}_e \gamma_\mu (1-\gamma_5)\psi_{\bar{\nu}}\right)$$

where:

G_F is the Fermi coupling constant,

$\psi_p, \psi_n, \psi_e, \psi_{\bar{\nu}}$ are the Dirac spinors for the proton, neutron, electron, and antineutrino, respectively,

γ^μ are the gamma matrices, and

γ_5 is the fifth gamma matrix, which accounts for chirality (a key feature of weak interactions).

The weak interaction involves the exchange of a virtual W boson (in modern theory), but in Fermi's theory, it is treated as a point-like interaction between fermions.

Fermi used **Fermi's Golden Rule** to calculate the decay rate of beta decay, which gives the probability of a neutron decaying per unit time. The formula for the decay rate Γ of a neutron in terms of the Fermi coupling constant G_F is:

$$\Gamma = \frac{G_F^2}{2\pi^3}|M|^2 p_e E_e$$

where:

$|M|^2$ is the matrix element squared, which depends on the specific decay process,

p_e is the momentum of the emitted electron,

E_e is the energy of the emitted electron.

The decay rate is related to the half-life $t_{1/2}$ of the neutron by:

$$t_{1/2} = \frac{\ln 2}{\Gamma}$$

Significance:

Fermi's theory of beta decay provides a theoretical framework to understand weak interactions and their role in radioactive processes. However, modern particle physics has expanded on this, incorporating the discovery of the W and Z bosons and the electroweak unification, which describe the weak interaction in a more complete manner (the Standard Model).

The energy released in **beta decay** comes from the difference in mass between the parent nucleus and the decay products. This energy is shared between the emitted beta particle (electron or positron), the neutrino (or antineutrino), and the daughter nucleus. The energy release in beta decay is as follows, There are two types of beta decay:

1. Beta-minus decay (β^- decay):

$$n \rightarrow p + e^- + \bar{\nu}_e$$

A neutron (n) decays into a proton (p), an electron (e^-), and an electron antineutrino ($\bar{\nu}_e$).

2. Beta-plus decay (β^+ decay):

$$p \rightarrow n + e^+ + \nu_e$$

A proton (p) decays into a neutron (n), a positron (e^+), and an electron neutrino (ν_e).

The total energy released in beta decay is primarily due to the mass difference between the parent particle and the decay products. This difference is converted into kinetic energy, which is shared among the emitted beta particle, the neutrino, and the daughter nucleus.

The total energy released, Q-value of the decay, is given by the equation:

$$Q = (m_{parent} - m_{daughter} - m_\beta - m_\nu)\,c^2$$

Where:

m_{parent} is the mass of the parent particle (or nucleus),

$m_{daughter}$ is the mass of the daughter particle (or nucleus),

m_β is the mass of the emitted electron (or positron),

m_ν is the mass of the emitted neutrino (which is very small, often approximated as zero),

c is the speed of light.

The Q-value represents the total energy released in the decay.

3. Explain the selection rules for Energy release in Beta-decay.

Ans. The energy released in beta decay follows certain **selection rules**, which govern the allowed transitions based on quantum mechanical principles like conservation of energy, angular momentum, and parity. The selection rules that govern beta decay energy release are,

1. Conservation of Energy: **Total energy** must be conserved in beta decay. The energy released in the decay, called the **Q-value** of the decay, is the difference between the total mass-energy of the

parent particle and the sum of the masses of the decay products (daughter nucleus, beta particle, and neutrino). The energy released Q, is given by

$$Q = (m_{parent} - m_{daughter} - m_\beta - m_\nu)c^2$$

where:

m_{parent} is the mass of the parent nucleus or particle,

$m_{daughter}$ is the mass of the daughter nucleus,

m_β is the mass of the emitted beta particle (electron or positron),

m_ν is the mass of the emitted neutrino (which is very small and often approximated as zero).

2. Conservation of Lepton Number:

a) In **beta-minus decay** (electron emission), the total **lepton number** must be conserved. A neutron (n) decays into a proton (p), an electron (e^-), and an electron antineutrino ($\bar{\nu}_e$).

i) The initial lepton number is 0 (for the neutron).

ii) The final lepton number is +1 from the electron and – 1 from the antineutrino, giving a net lepton number of 0. This ensures **lepton number conservation.**

b) In **beta-plus decay** (positron emission), the **positron** (which has lepton number – 1) is emitted along with a neutrino (which has lepton number +1), preserving the lepton number.

c) This selection rule is a direct consequence of **the law of conservation of lepton number**.

3. Conservation of Angular Momentum (Spin):

a) **Total angular momentum** must be conserved in beta decay. The total angular momentum of the system (which includes the parent nucleus, emitted electron or positron, and neutrino) must remain unchanged before and after the decay.

b) For example, in **beta-minus decay,** the decay of a neutron (which has spin $\frac{1}{2}$) to a proton (which also has spin $\frac{1}{2}$) and the emission of an electron (spin $\frac{1}{2}$) and an antineutrino (also spin $\frac{1}{2}$) must satisfy the requirement that the total spin before and after decay is conserved.

4. **Conservation of Parity:**

a) **Parity** is a symmetry property related to the spatial inversion (reflection) of the system. In the case of weak interactions, such as beta decay, **parity is not conserved.** This is a fundamental feature of the weak force, which distinguishes it from electromagnetic and strong forces.

b) In beta decay, the **weak interaction** violates parity conservation, meaning the parent and daughter states can have different parity. For example, in beta-minus decay, the initial state of the system (neutron) and the final state (proton and electron) can have opposite parity due to the weak interaction's violation of parity.

c) This violation is a crucial feature of the weak decay process, and there is no strict **parity selection rule** for beta decay in terms of parity conservation. However, angular momentum considerations still govern the decay process, and certain allowed transitions depend on the quantum mechanical wave functions of the particles involved.

5. **Allowed Transitions (Spin and Isospin Rules):**

a) Isospin (Isotopic Spin) Selection Rule: The weak interaction (responsible for beta decay) does not change the isospin of the decaying particle. For example, in **beta-minus decay**, a neutron (isospin $I = \frac{1}{2}$) transforms into a proton (isospin $I = \frac{1}{2}$), which is an allowed transition.

b) Spin Selection Rule: In general, beta decay involves the emission of an electron (or positron) and a neutrino (or antineutrino) with total spin **$S = 1$,** and the initial and final spins must add up in such a way that angular momentum is conserved. This leads to specific selection rules for the possible states of the parent and daughter particles.

6. **Energy Sharing Between the Particles:**

a) In beta decay, the energy released is shared between the **beta particle** (electron or positron), the **neutrino,** and the **daughter nucleus.** The distribution of this energy follows the kinematics of the decay process, ensuring that the total energy is conserved.

b) The electron's energy spectrum is continuous because the energy is shared between the electron and the neutrino, and both have unknown (and variable) energy contributions. The daughter nucleus usually takes only a small fraction of the energy in the form of recoil, given its much larger mass compared to the electron and neutrino.

7. **Forbidden Transitions (Selection Rules and Exceptions):**

a) **Forbidden transitions** in beta decay occur when certain quantum numbers (like orbital angular momentum) are not allowed by the selection rules. For example:

i) If the parent and daughter nuclei have the same quantum state, the transition is allowed and typically occurs with a relatively high probability.

ii) **First forbidden transitions** occur when there is a change in orbital angular momentum between the parent and daughter states, leading to a **lower probability of decay.**

iii) The strength of these transitions can be calculated, and higher-order (forbidden) transitions are less probable than allowed transitions.

4. Explain the principles of Electrostatic and Electro dynamical accelerators.

Ans. Electrostatic accelerators and **electrodynamics accelerators** are two broad categories of particle accelerators that use different methods to accelerate charged particles. They both utilize electric and magnetic fields, but they operate on distinct principles and are used for different types of applications in science, medicine, and industry.

1. **Electrostatic Accelerators:**

Electrostatic accelerators use static (non-varying) electric fields to accelerate charged particles. These accelerators typically use high-voltage sources to generate strong electric fields that accelerate particles in a linear direction.

Principle of Operation: Electrostatic accelerators use static (non-varying) electric fields to accelerate charged particles. The electric field is created by applying a high voltage to two electrodes, which generate a potential difference between them. When a charged particle is placed in this electric field, it experiences a force proportional to the field strength and moves towards the electrode of opposite charge.

The charged particle gains energy as it moves through the field. The amount of energy gained depends on the voltage applied and the charge of the particle.

Ex: Van de Graaff Accelerator, Tandem Accelerator, Cockcroft-Walton Accelerator etc.

Key Features:

High Voltage: Electrostatic accelerators rely on high-voltage sources to create the electric fields needed to accelerate particles.

Limited Particle Energy: The energy a particle can achieve is constrained by the voltage that can be applied. Thus, electrostatic accelerators are typically used for low- to medium-energy applications.

Simple Design: These accelerators are relatively simple in design and are more compact than some other types of accelerators.

Applications:

Medical: Used for **radiation therapy** to treat cancer by generating high-energy particles (electrons or protons).

Nuclear Physics: Used in experiments to study the structure of nuclei, particle collisions, and isotopic analysis.

Isotope Production: For the production of isotopes used in medical imaging and treatment.

2. Electrodynamics Accelerators:

Electrodynamics accelerators, also known as **dynamical accelerators**, use **time-varying electric fields** and **magnetic fields** to accelerate particles. These accelerators often use oscillating electric fields (such as in RF cavities) to impart energy to the particles as they travel through the accelerator.

Principle of Operation:

Magnetic and Electric Fields: Electrodynamics accelerators use alternating electric fields and magnetic fields to accelerate particles. In these devices, the magnetic field is used to steer and focus the particles, while the electric field is responsible for the actual acceleration.

Lorentz Force: The charged particle experiences a force when it moves through these fields. The electric field accelerates the particle, while the magnetic field bends the trajectory of the particle, allowing for circular or spiral paths, which is useful for keeping particles on track in the accelerator.

Ex: Cyclotron, Synchrotron, Linear Accelerators (Linacs), Free Electron Lasers (FELs).

Key Features:

Time-Varying Electric and Magnetic Fields: Electrodynamics accelerators rely on **oscillating electric fields** (from RF cavities) and **magnetic fields** (for steering) to accelerate particles.

High Energy: These accelerators can achieve **very high particle energies**, making them suitable for high-energy physics experiments, and particle and nuclear research.

Large Scale: These accelerators are typically much larger and more complex than electrostatic accelerators.

Applications:

High-Energy Physics: Used in major particle physics experiments (e.g., the **Large Hadron Collider (LHC)** at CERN) to investigate the fundamental structure of matter.

Medical: High-energy linacs and cyclotrons are used in **radiation therapy** to treat cancers and in **medical imaging** (e.g., producing isotopes for PET scans).

Material Science: Synchrotrons and other electrodynamic accelerators are used for material testing, surface studies, and X-ray diffraction.

5. Explain the construction and working of Cyclotron.

Ans. A **Cyclotron** is a type of **particle accelerator** that uses a **magnetic field** to bend the path of charged particles, and an **electric field** to accelerate them in a circular or spiral trajectory.

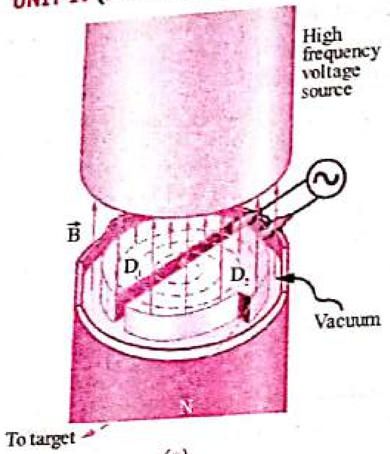
Construction: The construction of a cyclotron involves the following components.

1. Magnetic Field (B): A strong, uniform magnetic field is created by a **circular electromagnet** that surrounds the accelerator. The magnetic field is typically perpendicular to the plane of motion of the particles, and is responsible for bending the particle's trajectory into a circular or spiral path.

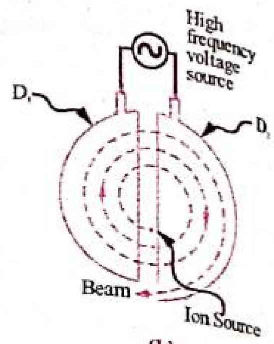
The magnetic field's strength determines the radius of the particle's circular path. The stronger the magnetic field, the smaller the radius for a given particle energy.

2. Dees (Electrodes): The **Dees D_1, D_2** are two **D-shaped electrodes** placed opposite each other within the magnetic field. These dees are placed inside a vacuum chamber. The dees alternate between being positively and negatively charged, creating an electric field between them. When the charged particle passes through the gap between the dees only, the electric field accelerates the particle.

3. Vacuum Chamber: The dees and the particles are contained within a vacuum chamber to avoid collisions with air molecules, which would reduce the energy of the particles.



(a)



(b)

4. High Voltage Power Supply: A high-voltage **radio frequency (RF) oscillator** is used to alternate the electric field in the dees. This alternating electric field accelerates the charged particles as they move from one dee to the other.

The frequency of the RF oscillator matches the time it takes for the particle to make a half-circle in the magnetic field, ensuring that the particles receive an energy boost every time they pass through the gap between the dees.

5. Ion Source: An **ion source** is used to generate the charged particles (ions) to be accelerated. The ions are injected into the cyclotron's center, where they are accelerated.

6. Extraction System: Once the particles reach the desired energy, they are extracted from the cyclotron and directed toward a **target** .This extraction is done by using a **deflector** or **electrostatic field** that diverts the particles toward the target.

7. Cooling System: A **cooling system** is needed to keep the cyclotron's components from overheating.

Working:

The positive ions emitted from the source will be accelerated in the gap towards the dee which is negative at that time. Let it be D_2. Since there is no electric field inside the dees, the positive ions move with constant velocity along circles of constant radius under the influence of magnetic field which is perpendicular to the dees. If by

the time the ions emerge from D_2, the polarity of the applied potential is reversed, (i.e., the dee D_1 now becomes negative), the positive ions will again face the negative dee and thus will be again accelerated by the field in the gap. Since their velocity is increased, they will now move through D_1 along circular arc of greater radius as shown in the figure. Here the time of passage to complete the semi circle in the dee remains the same as in D_2. If the time of travel in D_1 is equal to half the time period of the oscillator voltage, the positive ions after coming from D_1 will find the reversed field and hence they are accelerated again in the gap D_1D_2. In this way, the positive ions move faster and faster moving in ever-expanding circles until they reach the outer edge of the dees where they are deflected by deflector plate and strike the target. Here it should be remembered that the time required for the positive ions to make one complete turn within dees is the same for all speeds and is equal to the time period of the oscillator.

Theory:

Consider a particle of mass m and charge q moves with a velocity V in the magnetic field of flux density B, then r be the radius of the circular path .

The magnetic force provides the **centripetal force** necessary for the circular motion. The equation for the force on the particle moving in a circle under the influence of a magnetic field is given by,

$$BqV=\frac{mV^2}{r} \Rightarrow r=\frac{mV}{Bq} \quad ......(1) \quad \text{and } V=\frac{Bqr}{m}$$

The particle travels in a circular orbit with radius r. The time taken to complete one full revolution or the period T is given by, $T=\frac{2\pi r}{V}$. Substituting the value of r from (1),

$$T=\frac{2\pi\left(\frac{mV}{Bq}\right)}{V} \Rightarrow T=\frac{2\pi m}{Bq}$$

The **orbital frequency** f is the reciprocal of the period T. Hence,

$$f=\frac{1}{T}=\frac{Bq}{2\pi m}$$

Hence, the orbital frequency depends only on the charge q of the particle, the magnetic field strength B, and the mass m of the particle. It is independent of the velocity of the particle.

Energy of a particle accelerated by cyclotron: The KE of the particle in an orbit of radius r is given by,

$KE = \frac{1}{2}mV^2$ We have $V = \frac{Bqr}{m}$

Hence, $KE = \frac{1}{2}m\left(\frac{Bqr}{m}\right)^2 \Rightarrow KE = \frac{B^2q^2r^2}{2m}$

6. Explain the construction and working of Synchrocyclotron.

Ans. Principle: The fundamental principle of a synchrocyclotron is very similar to that of the **cyclotron.** In cyclotron, the frequency of the applied electric field is constant. But in a synchrocyclotron, the frequency of the **accelerating electric field** is **adjusted** in accordance with the particle's increasing velocity.

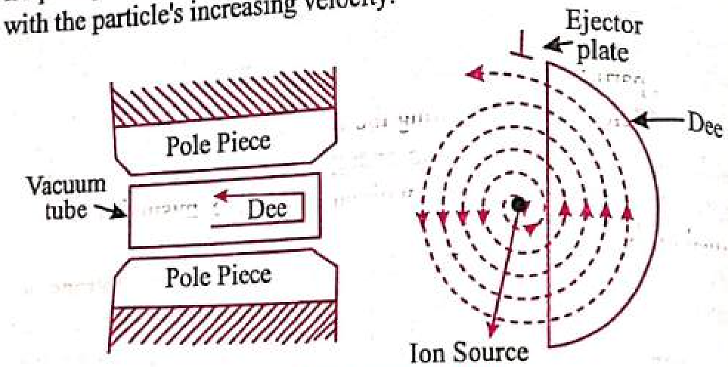


Fig: Synchro-cyclotron

Construction of Synchro-cyclotron:

The Berkeley synchrocyclotron is shown in Fig. It has a huge magnet with a pole diameter of 4.7 m. and weighing 4000 tons. The modulation frequency is 120 Hz. This is produced by a large rotating condenser that forms the part of capacitance in r f oscillator circuit. To reduce the scattering of ions by residual gas molecules, the pressure in the chamber is reduced to about 10^{-6} cm of mercury. Each ion makes over 10^4 revolutions before attaining maximum energy. Here a single dee is imployed. This feature makes the ion beam much more accessible for experiment. The ejector plate with a high positive voltage pulse deflects the ion groups on to the target to be bombarded.

Synchro-cyclotron can accelerate deuterons to 200 MeV, α-particles to 400 MeV and protons to 350 MeV.

Principle of Frequency Modulation in a Synchrocyclotron:

As the particle's velocity increases, its **orbital frequency** decreases because its relativistic mass increases. The synchrocyclotron adjusts the frequency of the electric field to stay in synchronization with this change, enabling continuous acceleration.

Synchrocyclotron modulates the frequency of the accelerating electric field to match the **relativistic increase in the mass** of the particle. This is done by **decreasing the frequency** of the accelerating voltage as the particle's velocity increases. Hence the particle stays in phase with the electric field, and it can continue to be accelerated efficiently.

Advantages of Synchrocyclotron:

1. Higher Energy Limits: The synchrocyclotron allows particles to be accelerated to **higher energies** than a traditional cyclotron, particularly for relativistic particles.

2. Efficiency: By adjusting the frequency of the electric field to match the particle's increasing energy, the synchrocyclotron allows for **continuous acceleration** without the phase mismatch seen in normal cyclotrons.

3. Better for High-Energy Physics: Synchrocyclotrons are particularly useful for **nuclear physics** experiments and **medical applications** like cancer therapy (via proton beams) due to their ability to achieve higher particle energies.

Limitations of Synchrocyclotron:

1. Complexity: The synchrocyclotron is more complex than a normal cyclotron due to the need for frequency modulation of the electric field. The RF system has to be precisely controlled and adjusted.

2. Size: Synchrocyclotrons can still be large, although they are more compact than linear accelerators or synchrotrons.

3. Energy Limitations: While the synchrocyclotron can accelerate particles to higher energies than a regular cyclotron, it still has practical limits on the maximum achievable energy. For extremely high energies, other types of accelerators, like **synchrotrons** or **linear accelerators**, are more efficient.

SHORT ANSWER QUESTIONS

7. Explain The Quantum Tunnelling Effect.

Ans. Gamow applied the concept of **quantum tunnelling** to explain alpha decay. In classical physics, an alpha particle inside a nucleus would need enough energy to overcome the Coulomb barrier (the electrostatic potential barrier created by the positive charge of the nucleus). However, in quantum mechanics, even if the energy of the alpha particle is lower than the barrier, there is a probability that it can "tunnel" through the barrier.

1. The Coulomb barrier arises due to the repulsion between the positively charged alpha particle and the positively charged nucleus (protons).
2. The alpha particle cannot escape via classical means because its kinetic energy is less than the height of the Coulomb barrier. But, in quantum mechanics, particles can pass through potential barriers with a nonzero probability, a phenomenon known as **quantum tunnelling**.

8. Explain the energy distribution in Beta Decay.

Ans. In beta decay, the total energy Q is divided between:

1. The beta particle (electron or positron),
2. The neutrino (or antineutrino),
3. The daughter nucleus (which typically receives a small amount of the energy due to recoil).

Beta-minus decay (β^-):

In **beta-minus decay,** the energy is distributed between the electron (or beta particle), the electron antineutrino, and the daughter nucleus. The energy of the emitted electron is variable, as the energy is shared between the electron and the neutrino.

1. The **energy of the electron (E_e)** and the **energy of the neutrino (E_ν)** are determined by the momentum and energy conservation laws. Since the neutrino has an extremely small mass, it carries away a significant portion of the energy in the form of momentum.
2. The **daughter nucleus** receives a very small amount of energy, typically in the form of recoil, due to its much larger mass compared to the electron and neutrino.

Beta-plus decay (β^+):

In **beta-plus decay,** the energy is distributed between the positron (or beta particle), the neutrino, and the daughter nucleus. The positron emitted in beta-plus decay is similar to an electron but carries a positive charge. Since a positron and an electron annihilate when they meet, the energy from beta-plus decay is often associated with a characteristic annihilation process, but for the energy release in the decay itself, the analysis is similar to beta-minus decay.

The energy is typically distributed in the following way:

1. The **electron** (or positron) carries away a significant portion of the energy, but its exact energy depends on the interaction dynamics.
2. The **neutrino** carries away the remainder of the energy. Since the neutrino is nearly massless, its energy is comparable to that of the electron, and it carries away momentum in a manner that ensures conservation of total energy and momentum.
3. The **daughter nucleus** (proton in the case of beta-minus decay) receives a very small fraction of the energy, due to its large mass.

9. Calculate the energy released in Beta Decay.

Ans. Consider the decay of a neutron (n) into a proton (p):

$$n \rightarrow p + e^- + \bar{\nu}_e$$

For this decay, the energy released (Q) can be calculated using the mass difference between the parent and decay products:

$$Q = (m_{neutron} - m_{proton} - m_{e^-})c^2$$

$m_{neutron} \approx 1.008665$ u,

$m_{proton} \approx 1.007276$ u,

$m_{e^-} \approx 0.0005486$ u.

The mass difference between the neutron and the proton-electron system is approximately:

$$Q = (1.008665 - 1.007276 - 0.0005486)\ u \times 931.5\ MeV/u$$

$$Q \approx 0.0008404\ u \times 931.5\ MeV/u \approx 0.783\ MeV$$

Thus, the energy released in the decay of a neutron is approximately 0.783 MeV. This energy is shared between the emitted electron (or beta particle), the antineutrino, and the recoil of the proton.

10. Briefly mention the selection rules in beta decay.

Ans. 1. Energy Conservation: The total energy (mass-energy) must be conserved.

2. Lepton Number Conservation: Lepton number is conserved (electrons/positrons and neutrinos/antineutrinos must balance).

3. Spin Conservation: Total angular momentum (spin) is conserved.

4. Parity Violation: Parity is violated in weak interactions, so no parity conservation is required.

5. Orbital Angular Momentum (*l*): In allowed transitions, there is no change in orbital angular momentum.

6. Isospin Conservation: The weak interaction conserves isospin, meaning the parent and daughter particles must have the same isospin.

7. Allowed vs. Forbidden Transitions: Allowed transitions occur when quantum numbers do not change significantly. Forbidden transitions involve changes in quantum numbers and are less probable.

These selection rules govern the nature of the beta decay process and determine the allowed transitions, energy spectra, and probabilities of various beta decay processes.

11. Give the differences between Electrostatic and Electro dynamical accelerators.

Ans.

Feature	Electrostatic Accelerators	Electrodynamics Accelerators
Type of Electric Field	Static (non-varying) electric fields	Time-varying electric fields (oscillating RF fields)
Type of Magnetic Field	Not used, except for steering (in Van de Graaff) .	Magnetic fields are used for steering (in cyclotrons, synchrotrons)
Particle Energy	Limited to low and medium energies	Can accelerate particles to very high energies
Design	Simple, compact	Complex, large-scale
Applications	Medical, nuclear physics, isotope production	High-energy physics, medical radiation therapy, synchrotron research
Examples	Van de Graaff, Tandem Accelerators, Cockcroft-Walton	Cyclotron, Synchrotron, Linac, Free Electron Laser

12. Give the Advantages and Disadvantages of Electrostatic accelerators.

Ans. Advantages:

Simple Design: Electrostatic accelerators are typically easier to build and maintain than electrodynamics accelerators.

High Voltage, Low Current: They can provide high voltage and are useful for applications requiring low current but high voltage, such as in ion implantation or small-scale nuclear physics experiments.

Disadvantages:

Limited Energy: The energy that can be imparted to particles is limited by the breakdown of materials (such as air) under high voltage and the practical limits of voltage application.

Size and Weight: While simpler than electromagnetic accelerators, they can still be quite large, especially in the case of high-voltage devices like the Van de Graaff generator.

13. Give the Advantages and Disadvantages of Electrodynamics accelerators.

Ans. Advantages:

Higher Energy: Electrodynamics accelerators can achieve much higher energies than electrostatic accelerators.

Continuous Acceleration: Electrodynamics accelerators can continuously accelerate particles, unlike electrostatic accelerators, which typically provide a one-time boost per particle.

Compact Design: Devices like cyclotrons and synchrotrons are relatively compact compared to electrostatic accelerators of similar energy.

Disadvantages:

Complexity: Electrodynamics accelerators are more complex to design, build, and maintain than electrostatic accelerators.

Cost: They tend to be more expensive, both in terms of construction and operation, due to the need for high-precision equipment and large magnetic fields.

Energy Loss: Synchrotrons and cyclotrons suffer from energy losses due to synchrotron radiation (for charged particles moving in circular paths), although modern technologies mitigate this issue.

14. Give the applications of Cyclotron.

Ans. Medical Applications:

Isotope Production: Cyclotrons are widely used to produce radioisotopes for medical imaging and treatment, such as those used in positron emission tomography (PET) scans.

Radiation Therapy: Cyclotrons are used to generate high-energy protons or heavy ions for cancer treatment, where charged particles are directed at tumors to destroy cancer cells with precision.

Nuclear Physics:

Cyclotrons are used to accelerate ions for nuclear research, including experiments that involve particle collisions, nuclear reactions, and the study of atomic nuclei.

Industry:

Cyclotrons can also be used for material science applications, such as irradiating materials to change their properties or test their durability under high-energy particle bombardment.

Research in Particle Physics:

Cyclotrons can be used in conjunction with other accelerators in particle accelerators for high-energy physics experiments, although they are typically limited to lower-energy experiments compared to synchrotrons and linear accelerators.

15. Give the limitations of Cyclotrons.

Ans. 1. Energy Limitation: Cyclotrons are limited in the energy they can impart to particles because the radius of the path increases as the particle gains energy, which eventually exceeds the physical size of the cyclotron. This limits the maximum energy that can be achieved.

2. Relativistic Effects: For very high-energy particles, relativistic effects become significant. As particles approach the speed of light, their mass increases, and they require more energy for further acceleration, making it difficult to continue using a simple cyclotron.

3. Size: Compared to other accelerators, large cyclotrons require significant space, especially for higher energies.

16. Briefly explain the construction of synchro cyclotron.

Ans.

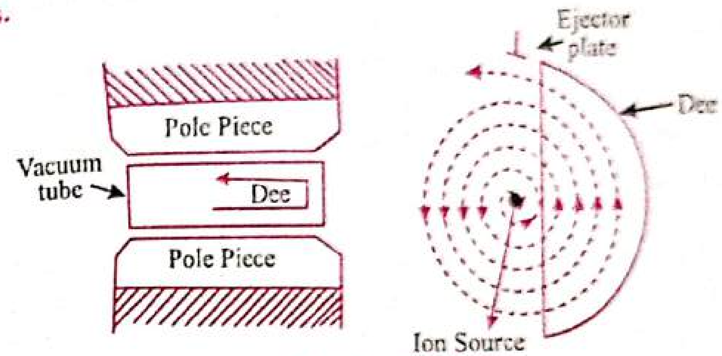


Fig. Synchro-cyclotron

Construction of Synchro-cyclotron:

The Berkeley synchrocyclotron is shown in Fig. It has a huge magnet with a pole diameter of 4.7 m. and weighing 4000 tons. The modulation frequency is 120 Hz. This is produced by a large rotating condenser that forms the part of capacitance in r f oscillator circuit. To reduce the scattering of ions by residual gas molecules, the pressure in the chamber is reduced to about 10^{-6} cm of mercury. Each ion makes over 10^4 revolutions before attaining maximum energy. Here a single dee is imployed. This feature makes the ion beam much more accessible for experiment. The ejector plate with a high positive voltage pulse deflects the ion groups on to the target to be bombarded.

Synchro-cyclotron can accelerate deuterons to 200 MeV, α-particles to 400 MeV and protons to 350 MeV.

17. Give the Advantages and limitations of Synchrocyclotron.

Ans. Advantages of Synchrocyclotron:

1. Higher Energy Limits: The synchrocyclotron allows particles to be accelerated to **higher energies** than a traditional cyclotron, particularly for relativistic particles.

2. Efficiency: By adjusting the frequency of the electric field to match the particle's increasing energy, the synchrocyclotron allows for **continuous acceleration** without the phase mismatch seen in normal cyclotrons.

3. Better for High-Energy Physics: Synchrocyclotrons are particularly useful for **nuclear physics** experiments and **medical applications** like cancer therapy (via proton beams) due to their ability to achieve higher particle energies.

Limitations of Synchrocyclotron:

1. Complexity: The synchrocyclotron is more complex than a normal cyclotron due to the need for frequency modulation of the electric field. The RF system has to be precisely controlled and adjusted.

2. Size: Synchrocyclotrons can still be large, although they are more compact than linear accelerators or synchrotrons.

3. Energy Limitations: While the synchrocyclotron can accelerate particles to higher energies than a regular cyclotron, it still has practical limits on the maximum achievable energy. For extremely high energies, other types of accelerators, like **synchrotrons** or **linear accelerators**, are more efficient.

18. Distinguish between Cyclotron and Synchrocyclotron.

Ans. 1. Electric Field Frequency:

Cyclotron: Uses a **constant frequency** for acceleration, which works for low-speed (non-relativistic) particles but leads to inefficiencies at high energies.

Synchrocyclotron: Adjusts the frequency of the electric field as the particle's velocity increases, compensating for **relativistic effects** and allowing higher energy acceleration.

2. Relativistic Effects:

Cyclotron: Ineffective for relativistic particles because the fixed frequency becomes out of sync with the particle's increasing speed.

Synchrocyclotron: Specifically designed to handle relativistic particles by varying the frequency, thus efficiently accelerating particles at relativistic speeds.

3. Energy Limits:

Cyclotron: Limited to relatively low energies (typically a few tens of MeV) due to the frequency mismatch at high velocities.

Synchrocyclotron: Can achieve **higher energies** (up to 100 MeV) by adjusting the frequency for relativistic particles.

4. Design Complexity:

Cyclotron: Simpler design, making it cheaper and easier to build, but with limitations at high energies.

Synchrocyclotron: More complex, as it requires a **variable frequency** system to match the particle's increasing velocity.

19. Give the applications of Synchrocyclotron.

Ans. Due to its ability to adjust the frequency of the accelerating electric field to match the increasing velocity of particles, synchrocyclotrons have found significant applications in a range of fields.

1. Nuclear Physics Research: It is used in generating high-energy particle d in nuclear reactions, neutron production, and radioactive isotope production. Synchrocyclotron is also involved in nuclear fission, fusion, and to study the behaviour of elementary particles.

2. Medical Applications: Synchrocyclotrons are used is in cancer therapy, proton therapy in radiation oncology.

Synchrocyclotrons are used to produce radioactive isotopes for medical imaging and treatments such as positron emission tomography (PET), single-photon emission computed tomography (SPECT), and radiation therapy.

3. Synchrocyclotrons are used in research in Material Science to study the effects of radiation damage on materials.

4. Isotope Production for Industry: Synchrocyclotrons can be used to produce radioisotopes for non-destructive testing, such as radiography to inspect materials for flaws or defects.

5. In the field of high-energy physics, synchrocyclotrons have been used as part of experimental setups for investigating the fundamental forces and particles.

6. Studying Nuclear Reactions: Synchrocyclotrons help researchers to study nuclear reactions. These studies contribute to our understanding of processes such as nuclear fusion, fission, and nuclear decay. Synchrocyclotrons are used in space research.

SOLVED PROBLEMS

20. In a cyclotron, the frequency applied to the dees is 8.6 mega cycle/sec. Calculate the magnetic field of induction required to accelerate the protons. (mass of proton = 1.79×10^{-27} kg).

Sol. We know that

$$f_0 = \frac{qB}{2\pi m} \quad \text{or} \quad B = \frac{2\pi m f_0}{q}$$

Substituting the given values, we have

$$B = \frac{2\times3.14\times(1.79\times10^{-27})\times(8.6\times10^{6})}{1.6\times10^{-19}} = 0.6043 \text{ Weber/m}^2.$$